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Significance of the "cut off-average grade-tonnage" sensitivity analysis in mining projects

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ABSTRACT

The sensitivity analysis applied to the grade-tonnage relationship in mining projects is a recognized stage in all existing quality standards. However, once presented, the results are hardly commented on or criticized. This study analyzes more than 50 projects grouped by copper and gold metallogenetic models which find their own characteristics in the tonnage evolution curves by varying the cut-off grade. In addition, when considering the categories of quality in information regarding resources, those which are classified as inferred usually denote a lack of quality. This study also offers an explanation about the behaviour of ore typologies with important resources and non-abrupt grade variations, with other typologies with well-declared anisotropies and strong structural controls.

Keywords: cut-off, mining quality standards, mining projects, resources qualification, sensitivity analysis technical reports.

Significado del análisis de sensibilidad "ley de corte-ley media-tonelaje" en un proyecto minero

RESUMEN

El Análisis de Sensibilidad referido a la relación ley-tonelaje en un proyecto de inversión minera resulta un paso reconocido en todos los estándares de calidad actuales. Sin embargo, después de presentarlo, apenas se comenta ni critican los resultados. El trabajo analiza más de 50 proyectos actuales agrupados por modelos metalogenéticos de cobre y oro, encontrando características propias en las curvas de evolución del tonelaje al variar la ley de corte. Además, al considerar las categorías de calidad de información en los recursos, los calificados como inferidos, suelen denotar la falta de esta calidad. También, el trabajo trata de encontrar una explicación de las diferencias que aparecen entre las tipologías que contienen un gran volumen de recursos y con variaciones no bruscas de sus leyes, con otras tipologías con anisotropías muy declaradas y fuertes controles estructurales.

Palabras clave: calificación recursos, estándares minería, proyectos mineros, informes técnicos.

Introduction

In a technical report on a mining investment viability project, the sensitivity analysis refers to the curves or functions that relate the average grade with the tonnage to the different cut-off grades. All this refers to a mineral deposit that is subjected to technical and economic analysis for final qualification. In the quality standards derived from the Australasian Joint Ore Reserves Committee (JORC), the concept of Grade Sensitivity Analysis is widely commented (AusIMM, 2012). In addition, it can be undertaken after the elaboration and acceptation of the block model and before the determination of the reserves, where the limiting factors are already included.

That is, in this section on the determination of reserves all types of geometry are acceptable, when exploitable (Block Model) and only with the limitation imposed by the cut-off grade and the dimensions assigned to the blocks. At this stage, the uncertainty related to the limit grade selection is already included.

A real example of an investment project showing the relationship between the cut-off grade with the average grade and the corresponding tonnage can be is seen in Figure 1.

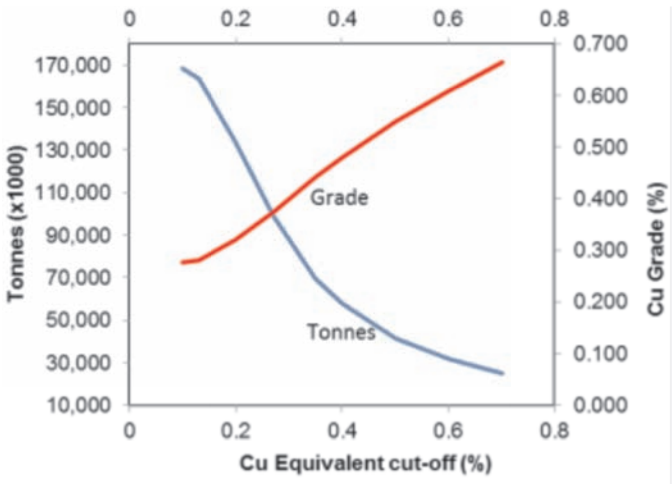

Figure 1. Example of the relationships between cut-off, average grade and reserves for a real example of investment project.
Figura 1. *Ejemplo de las relaciones entre corte, calificación promedio y reservas para un ejemplo real de proyecto de inversión.*

In the reserve definition phase, the usual model will incorporate the inaccuracies derived from the application of its main components: average grade, thickness and price assigned to the cut-off grade. All this is only reduced by the division of the solid in natural domains.

Although the grade-tonnage curve is important for most economic assessments, it is appropriate to remember that the model does not consider continuity between blocks. The approximation of the grade-tonnage curve to reality depends on natural parameters such as geology and geochemistry, as well as geometric distribution of the deposit. Usually, the more variable the grades are the more complex the geometry is and the less reliable the curve is (Silva and Soares, 2002).

Shortcomings in technical reports

Frequently a technical report does not always incorporate a true sensitivity analysis with respect to the grade-tonnage relationship. Sometimes, it only provides answers to two or three cut-off scenarios close to their most probable value. In addition, a curve or table of cut-off variations is never linked to metal price. Therefore, this causes a loss of very useful information to better understand the risk related to market variations.

The importance in the optimum cut-off selection referring to the grade-tonnage curve distribution, accompanies the mining project success throughout its operation. This analysis can be significant in maximizing the net present value (NPV) of the project and, in turn, in minimizing the negative environmental impact (Osanloo, 2008) that pursues mining today.

Technical reports rarely incorporate these comments on tables or sensitivity curves or, at best, only a few inconsequential comments appear. The effort is not well understood if it is not accompanied by any consequence regarding the morphology of the calculated functions. Now, however, the dedication to this aspect should not be very expensive during the analysis of the block model.

Background on sensitivity functions

Years ago, it was not difficult to find references to the relationship between the deposit average grades and their corresponding tonnage, as a minimum mineral grade should be considered to be able to economically work the deposit. Thus, Manteca (1993) indicates that when studying the relationship between the accumulated tonnages above different ranges of grades (cut-off grades) it is observed that the regression quality between most values and others is good. This relationship is an exponential function and is formalized by Lasky’s law, which is based on a lognormal model that reproduces the relationships between the grades and the corresponding accumulated tonnages.

$$m_c = K_t - k_2 l_n T_c$$

$$T_c = \frac{K_t}{k_2} \cdot e^{-m_{ck_2}}$$

where: m_c = grade average above a cut-off grade “c”, T_c = tonnage above a cut-off grade “c”.

The tonnage of ore accumulated above the cut-off grade increases geometrically as the cut-off law decreases arithmetically.

Lasky established this equation for the 10 largest copper porphyries in the United States at that time, and later verified it for deposits of other very different substances, such as Fe, Ni and Au (Lasky, 1950). The purpose of Lasky's law was to help the mine engineers to estimate the recoverable reserves in a given deposit. Initially, it was only applicable to copper porphyries. Musgrove (1971) verified the functionality of Lasky's law in Pb deposits, out of a total of 58 deposits, which confirmed that the relationship was also verified, not only in the mine blocks but also in the entire deposits. Subsequently, Singer and Young (1980) analyzed the Lasky relationship for 165 porphyry copper deposits. The arithmetic-geometric ratio of Lasky, or *Lasky's Law* as it was later recognized, provides different attributes of mineral resources into a cumulative grade-tonnage curve. De Young (1981) considered the Lasky relationship as consistent for studies relating average grades with tonnages. Nevertheless, there were misinterpretations in later studies about the available metal expected while the grade decreases. The projection to lower grades, out of the limit, is compromised, because the mathematical formulations predict impossible situations.

Lasky's law has also been successfully applied to predict the response of the natural stock of mineral raw materials to a supposed and inevitable current depletion of deposits. In addition, at the time it also provoked an interest in deepening and improving this relationship (eg. Gerst, 2008).

Previous considerations

At present, and after the ease of calculation provided by the block model, the possibilities of analysis have increased greatly allowing the verification of these ambiguous relationships in a more statistical and imaginative way. The focus of this study is to take advantage of the relationship suggested by Lasky from a more real or practical point of view. In this study we seek the use of the relationship provided by the block model to a first understanding on its effect on a mineral deposit. Then, it predicts the risk associated to volatility in metal price. This should be transferred into changes in the cut-off grades adopted at the beginning of the exploitation project.

The block model is a tool that begins with the consideration of explored resources and works strictly on geological basis (the solid or wireframe). Only the considerations of the quality of the information (drilling density and others) are reasons for the acceptance or not regarding the deposit. This is the case of the qualification as inferred resources, referring to those that still have a low information quality. Their consideration as indicated or measured resources, when the available information demonstrates its continuity, is the starting point towards reserve categories. This case is achieved by applying the "limiting factors". When talking about resources, quality refers to the degree of mineral-body continuity assurance captured in the samples.

In addition, the block model is affected by the geometrical dimensions of the chosen unit, usually linked to technical operation considerations. Once all these steps have been overcome, the model is ready to be the object of a fundamental selection. It is the cut-off grade, which is sometimes somewhat improvised. In any case those blocks that equal or surpass it will be selected. Given the variability of the many factors involved, particularly the price of metals, it is very interesting to know the transcendence of the variation in that limit grade. Hence, the opportunity for the grade-tonnage-metal content curves is found.

Sensitivity analysis in grade-tonnage curves

The grade-tonnage curves are often complemented by the metal content graph for each selected cut-off. In a technical report, their representation is justified by the intention to cover the future contingencies regarding the assignment of the basic parameters (metal price, operation costs, etc.).

The reasons to perform this analysis would be as follows:

- As a forecast of future contingency
- To build alarms and acceptance limits
- As a procedure to be included in the risk analysis
- To begin the elaboration of an anticipatory group measures of change:
    - more brownfield mining research
    - a cost reduction strategy
    - an improvement in plant and mine performances.

Despite this, in a technical report, it is rare where the relationship between the cut-off magnitudes, average grade, tonnage and metal content is analyzed.

Working methodology

The research methodology consists of the data collection on variation in resources and their average grades when the cut-off of real mining projects varies. The sources of the information have been the technical reports of recognized quality standards, especially NI 43.101. In these reports, these data appear in the form of tables and as curves included in the same representation (Fig. 2).

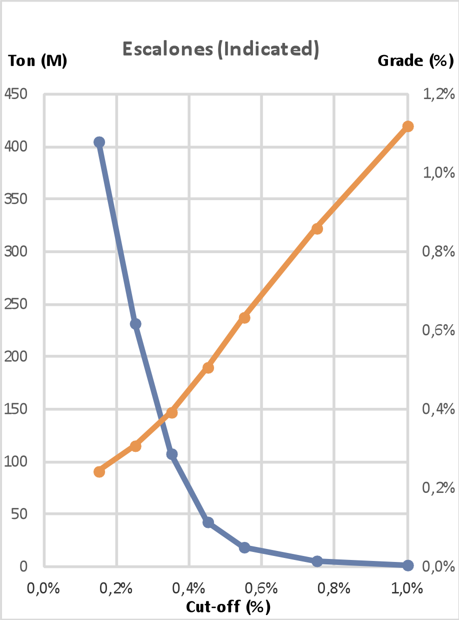

Figure 2. Grade-Tonnage-Cut-off curve representation of Escalones copper deposit, showing actual values.
Figura 2. *Curva ley-tonelaje-cut-off del yacimiento de pórfidos cupríferos de Escalones, mostrando sus valores actuales.*

This form of presentation does not allow direct comparison between curves, since the axe scales are so unlikely (Figs. 1 and 2) to be needed to represent very different magnitudes. So, the data that make up the variation curves were normalized, referring to them as a percentage to the maximum variation (Fig. 3 to Fig. 8).

In this way, there are two series of available graphics. The first set of curves defines the sensitivity in true magnitude of the average grade and tonnage determined by a variable cut-off. If a comparison with another project is desired, it would be sufficient to recognize a module applicable to the cut-off (e.g. 0.1% Cu or 0.1 gAu/t) to obtain the variation in tons or grade of the chosen new deposit. This, however, would not allow a comparison with other projects, since they would depend on the overall deposit magnitude. However normalization informs about the distribution of variations and, therefore, it will be studied by linking it to other characteristics, such as the genetic model of the deposit.

Furthermore, normalization tends to smooth the morphological variations, especially at the extremes. Possibly, in these extreme positions that deviate from the most probable ones for the project, errors or the very particular geometry situations of each deposit can also occur.

Information provided for the classification of indicated-measured resources versus inferred resources

In the porphyry copper model, when comparing examples of variations in its grade-tonnage curves related with its cut-off grades, in general, variations normalized to 100% present very similar forms (Fig. 3). To be more accurate, a trend to the spherical model can be observed in the curves that represent the values with the inferred category. This also happens with respect to the curves of indicated resources. The concept of spherical variation of tonnage refers to a model with a uniform mineral grade gradation towards the centre of the sphere that causes an exponential volume variation. This is understandable, since the indicated and measured resources qualification incorporates more details that do, either by smoothing the tonnage curve or even decreasing the variation in values corresponding to lower cut-offs.

On the other hand, the tonnage curve of the inferred resources presents a pronounced concavity outwards of the quadrant formed by the graph axes. However, in the indicated resources, this does not happen so strongly. This variation suggests more complex geometric models, diverging from their spheroidal model more than one core of higher grades. It would also be explained by a pronounced anisotropy in the grade distribution.

From this, there is the opportunity of deducing some estimates when considering the inferred resources (analyzing the similarity of their variations). This would be a new criterion to show off their potentiality. Currently, however, inferred resources are not considered in any technical report.

Two very defined domains: oxidized gold mineral and sulphurized fresh mineral

In gold deposits, sulphide ores are frequently oxidized near the surface and this causes a noticeable difference in the gold recovery process. Generally, this fact improves the recovery ratio and reduces production costs. We have focused on orogenic gold deposits in this study, typology widely present all over the world.

In this case, morphology of the sensitivity curves is very similar, highlighting a tendency of the oxidized ore to the globular or spherical model. This occurs with small variations that would result in an outwards concavity. It is more pronounced (although not definitive) in the deposit areas where the sulphides (which partly contain gold) remain unaltered (Fig. 4).

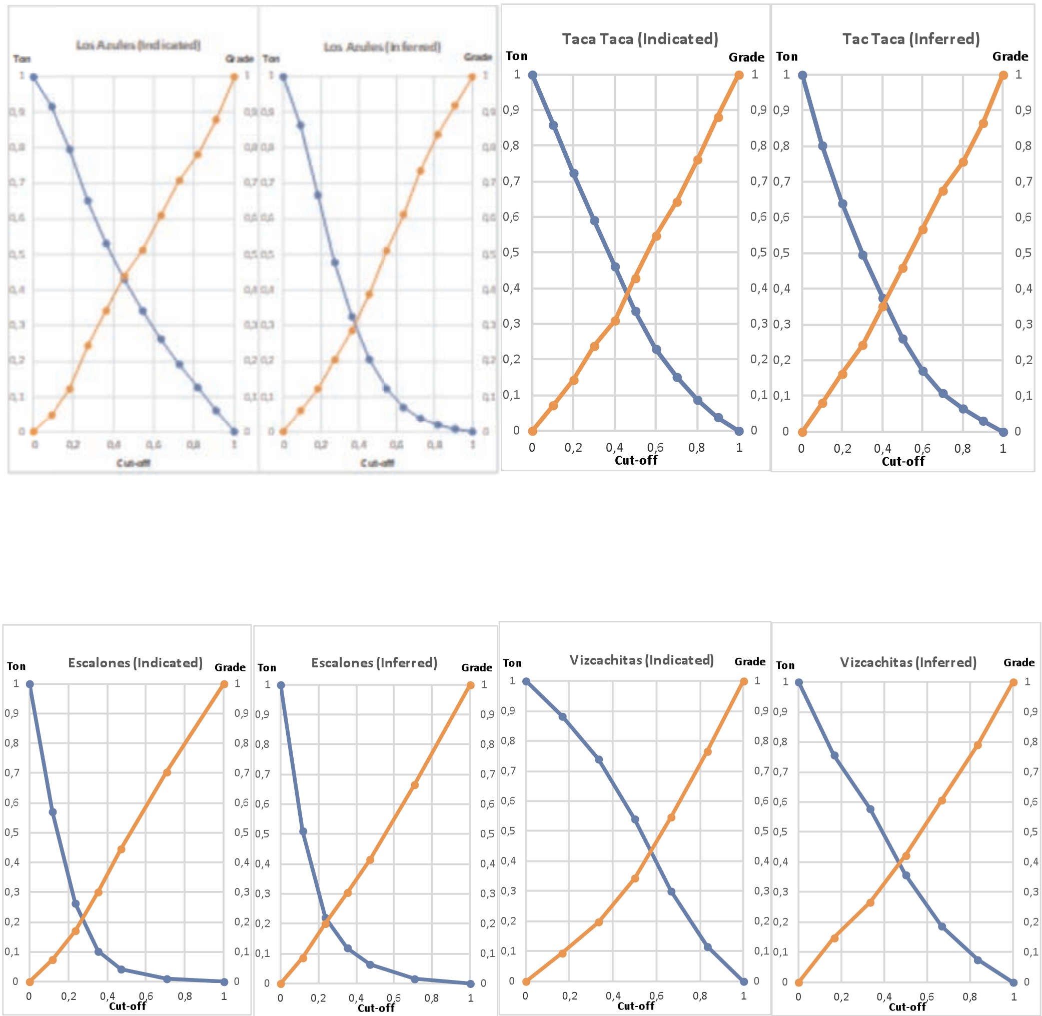

Figure 3. Grade-tonnage representation of resources classified as indicated and inferred in porphyry copper deposits.
Figura 3. *Curvas ley-tonelaje de los recursos clasificados como indicados e inferidos en depósitos de pórfidos cupríferos.*

Differences related to the genetic model

Undoubtedly, models help to explain individual cases through the contribution of common characteristics and with the idealization of the formation process of the mineral deposits. So, three main types widely present in the five continents have been chosen. They are producing significant quantities of two metals, copper and gold. In addition, we have chosen examples located to the same model in different world regions.

It must be borne in mind that, in the genetic model, the mineralization is carried out on a geological support, and this has a very specific geometric representation. Thus, in porphyry copper deposit, the fundamental characteristics that influence its economic use are the morphology (from cylindrical to spherical) together with its huge size. In addition, other characteristics linked to the concept of porphyry are its low grade and the low production cost. This is due to the use of large machinery and, above all, the low removal rates. That is to say, the intense exploitation that has taken place in recent years has substantially reduced its grade but not so much its tonnage. In fact, adaptations have already begun: underground mining of high productivity and low cost, underground mining in areas of high copper grade, productive intensity with low grade minerals and others.

The grade-tonnage curves taken as a whole and normalized to 100% of their maximum values, are grouped in a very defined linear way in terms of variations in their average grades. However, with respect to their tonnages, linked to the cut-off, they appe-

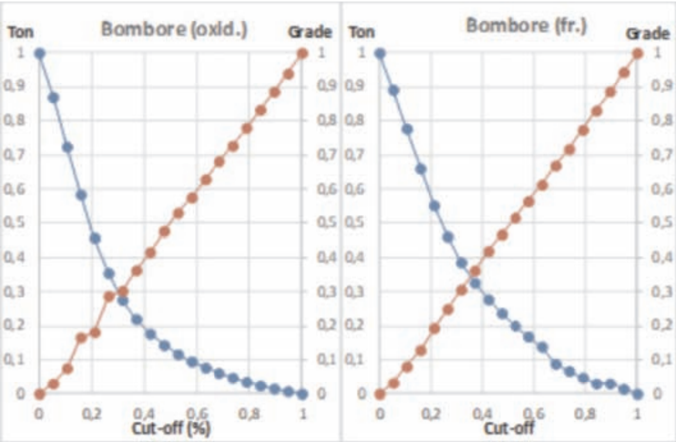

Figure 4. Grade-tonnage representation according to the cut-off for two mineralization types: oxidized and unaltered sulfides. "Orogenic gold deposit type" Project.

Figura 4. *Representación de la relación ley-tonelaje según la ley de corte para dos tipos de mineralización: sulfuros oxidados y los no alterados. Un proyecto sobre un depósito mineral del "tipo oro orogénico".*

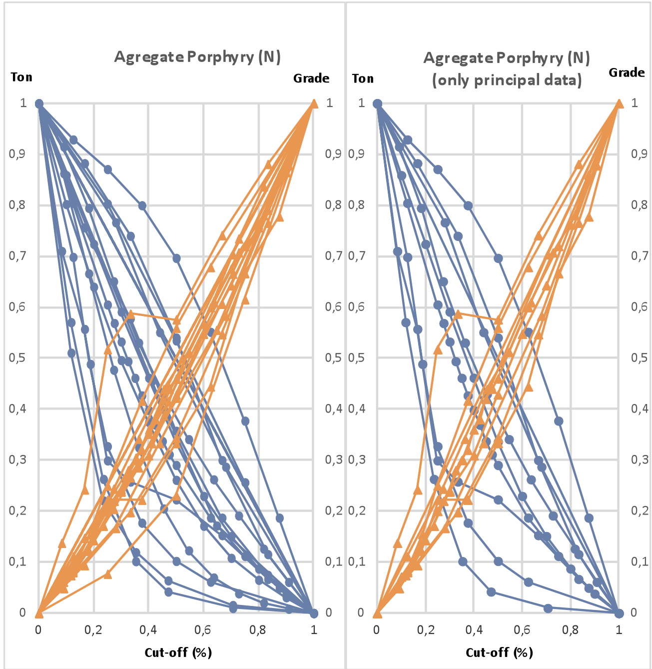

Figure 5. Aggregation of grade-tonnage curves corresponding to porphyritic copper deposits examples.

Figura 5. *Curvas ley-tonelaje correspondientes a los depósitos de cobre porfídico.*

ar predominantly with a pronounced concavity outwards in the diagram (Fig. 5). That is to say, they indicate a proper large volume behaviour and spherical or cylindrical geometry. However, there are also some cases with little initial slope and finally an increase in slope or sensitivity. As previously argued, this may be related to greater complexity due to several high-grade cores or high degree of anisotropy.

The copper deposits of the sediment hosted type (SH), had been partly forgotten in some parts of our planet due to the geo-political conditions. Both the discrete thicknesses of their ore-bearing bodies, compared to their longitudinal development and the fact that they are located in orogenic zones due to their age, result in its predominantly underground exploitation.

However, the good wall definition (low dilution at startup), the continuity of their mineralized bodies and, above all, their average grade, help their typically underground exploitation. Due to their variable geometry, it will often be necessary to change the operating method, from room and pillars in near horizontal deposits to sub-levels for the deeper ones. Their mineralogy, with sulphides richer in copper than chalcopyrite, produces high copper grade in concentrates. This improves the net smelting return (NSR) of the concentrate. The presence of other high unit value metals, especially cobalt and silver, helps the final economic results.

In the case of SH copper deposits, there are strong variations in the evolution of the average grade related to the cut-off. Its explanation must be sought, both in a great variety of situations within the same lithological control and in the errors inherent to selective sampling. This same effect is manifested in the tonnage linked to the average grade. The differences in the evolution curves are also very important, presenting both the effects of the spherical geometric model and gradual grade variation, as well as the presence of strong anisotropies in the deposit geochemistry (Fig. 6).

Orogenic gold deposits have been some of the most successful gold appearance models in recent years. Although it is a very broad model that includes many genetic disparities, the ores show very clear structural control. This results in narrow deformed zones in the fragile structural domain, that is, very broken areas with vetiform or brechoid morphology and verticalized position. For this reason, in most deposits its use is conditioned to underground exploitation. In the open-pit case it works with very high stripping ratios. This means that it is only possible to exploit those deposits with a relatively high gold grade.

The aggregate curves of orogenic gold deposits show a great coherence (Figure 7). The grade variations are manifested in a linear manner without great

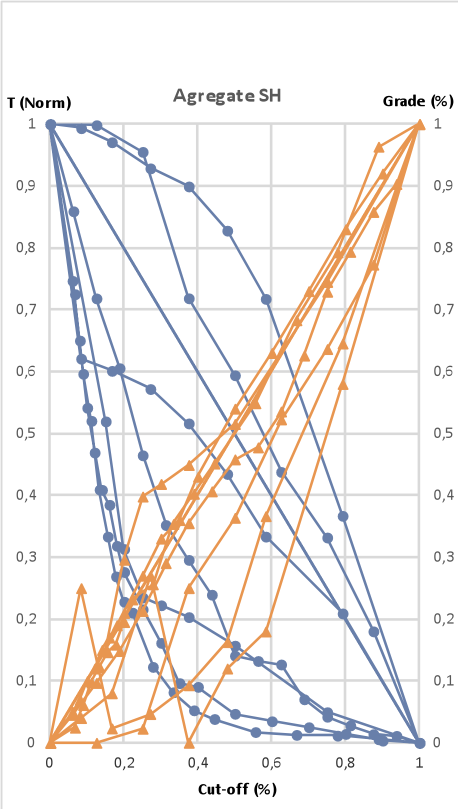

Figure 6. Grade-tonnage curves representing standardized variations of SH (Sediment Hosted type) copper mineral deposits.
Figura 6. *Curvas ley-tonelaje que representan variaciones estandarizadas de depósitos minerales de cobre de tipo SH (Sediment Hosted).*

exceptions and the tonnages linked to them always have an outwards concavity. The oxidized zones at the near surface domains of some cases are confused with the unaltered sulphide zones, demonstrating the conservative behaviour of precious metals once their sulphide support has oxidized. It is also striking that, as a whole, the behaviour is not too anisotropic or with specific foci, although structural control is the dominant feature of this type of deposit.

Cut-off grade and the domains of variation

As in the tonnage curve there are two sensitivity domains regarding the cut-off grade, it is interesting to relate at a given moment the slope change with the cut-off of the project. This is the one calculated from the circumstances of the moment. In the porphyry copper model, which is practically in all projects, the design cut-off is found before the slope change in the tonnage curve. Its average value is 65% of the inflection point. However, in orogenic gold projects, the calculated or chosen cut-off may be after the change in sensitivity and, in addition, the discrepancies can be enormous. The cut-off reflects the circumstances of the moment, especially those related to the metal markets and, to a certain extent, cannot be changed. However, the selection of a cut-off grade in the stretch curve section allows for less variation or a more stable position when there are strong variations in the

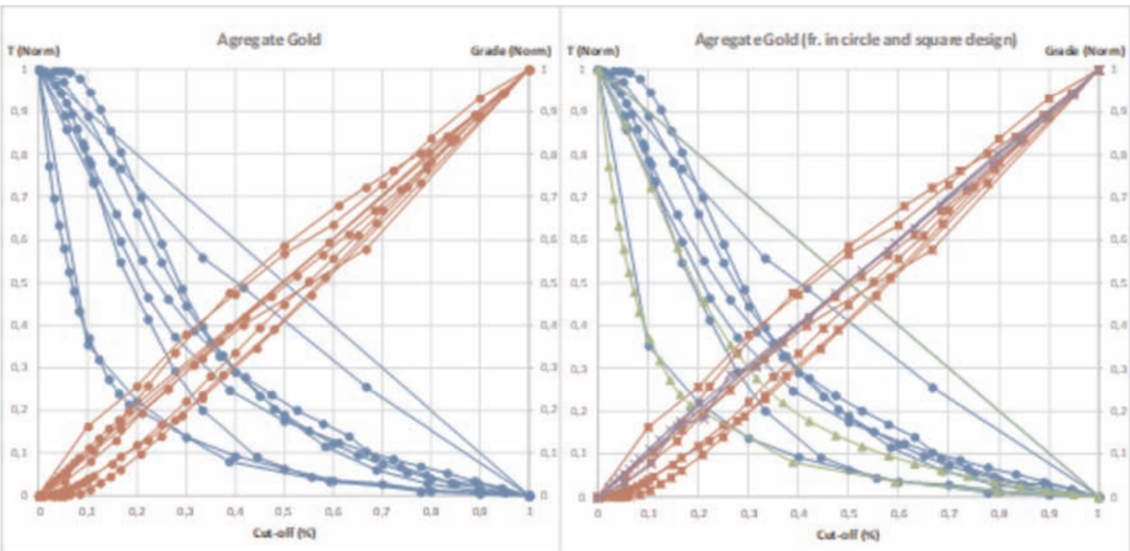

Figure 7. Curves representing standardized variations of grade-tonnage in “orogenic gold” mineral deposits. The gold curves in green contained in the tonnage valued have also been included.
Figura 7. *Curvas que representan variaciones estandarizadas de ley-tonelaje en depósitos minerales tipo “oro orogénico”. También se han incluido las curvas en color verde referentes al oro contenido en el tonelaje valorado.*

cut-off values, especially driven by the volatile markets.

When the deposit cut-off varies, the behaviour of gold resources is easily predictable. This is due to the reduction of average grade variations or the unitary content in the linear progression. As an example, Figure 8 shows how the metal contained curves mask the variations in tonnage for two different mines of the same deposit typology. Therefore, the construction of the metal contained diagrams does not inform us about the intensity of variations suffered as the cut-off changes. It would only inform us vaguely about its concavity or convexity, always in a much-smoothed way.

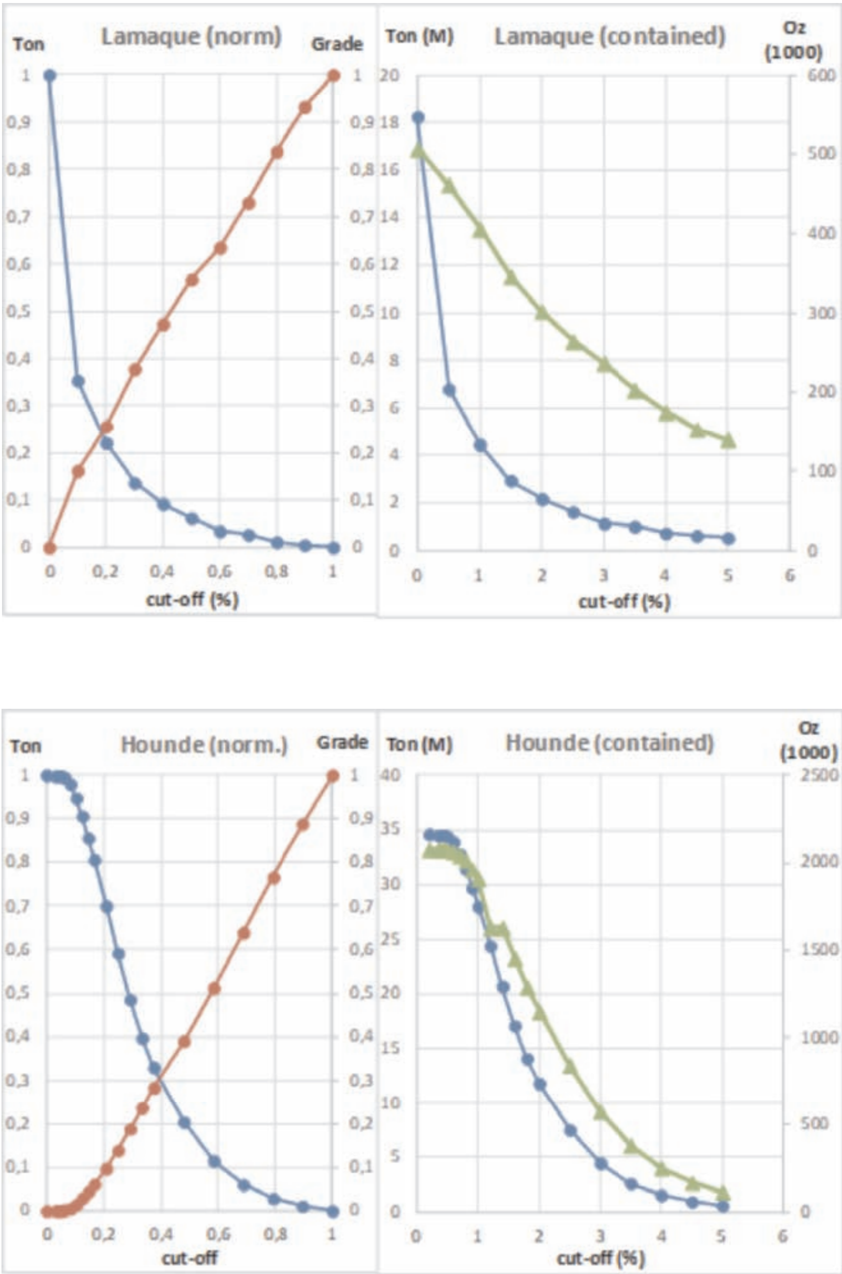

Figure 8. Representation of tonnage sensitivity and its value in contained gold (triangles) when the limit grade varies. Two deposits typology correspond to "orogenic gold".
Figura 8. *Representación de la sensibilidad del tonelaje y su valor en oro contenido (triángulos) cuando varía el cut-off. La tipología de dos depósitos corresponde a la de "oro orogénico".*

Discussion and conclusions

In general, the analysis of the grade-tonnage sensitivity is never commented on, although a lot of information can be obtained from it. Naturally, the most interesting cut-off interval is really located around the cut-off that has been chosen in the project, since, in a reasonable period of time, the magnitude of the variations in the metal prices that have an impact on the cut-off it is not usually especially significant.

By definition, the cut-off grade is the metal concentration that gives us the production operating costs. For this value, the curve of its average grade variation tells us about the potentiality of unit wealth creation in which the production of the project is valued. Generally, that is the ton moved and treated in the concentrator. The difference between the two grades is related to the operating margin that, by relating it to the possible exploitation tonnage or the point in the tonnage curve, will produce the information to appreciate the economic availability to cover the investment and remunerate the invested capital.

For all these reasons, the first interpretation refers to the acceptance degree of the genetic model that carries with it the characteristics related to the sensitivity of the grade-tonnage relationship.

If we look for a good behaviour of a mining investment project facing the inexcusable variations of the metal prices, it is interesting that, although the grade varies enormously, the tonnage varies little. In this way, the project can generate the economic flow capable of covering the fixed costs. In addition, in the case of a new project, the operating margin should be able to address the investment. Neither should we forget the available tonnage, so that its reduction does not affect the life of the project or the production capacity. Part of the questions raised can be answered in the grade-tonnage curves.

Summarizing the objectives and their compliance, the use of the sufficiently normalized variation curves has allowed us to:

- rate the project regarding its risk or sensitivity to failure.
- group the results by typology: progress, and economic and metalogenetic project characterization.
- define the best usable parts of the curve, within a risk situation.
- relate the indicators with the deposit geometry and with the defined tonnage.

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