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A quasi three-dimensional model for the simulation of seawater intrusion into coastal aquifers

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ABSTRACT

A finite element software based on a quasi three-dimensional model to simulate seawater intrusion in coastal aquifers has been established. The mathematical model is the same as that developed by Sorek *et al.* (2001) however, in this study we have used the finite element method with stabilisation of the convection-diffusion equation by the SUPG method (Streamline Upwind Petrov Galerkin). The code is applied to an unconfined aquifer. The hydraulic head and chlorine concentration contour maps, which also position the interface, are presented.

Key words: coastal aquifer, intrusion, modeling, quasi three-dimensional, sea water.

Un modelo cuasi-tridimensional para la simulación de intrusión marina en acuíferos costeros

RESUMEN

Este artículo trata sobre un programa computacional de elementos finitos basado en un modelo cuasi-tridimensional para simular la intrusión marina en acuíferos costeros. El modelo matemático es el mismo al desarrollado por Sorek et al. *(2001). Sin embargo, en este trabajo se ha utilizado el método de elementos finitos con estabilización de la ecuación de convección-difusión por el método SUPG (Streamline Upwind Petrov Galerkin). El código se ha aplicado a un acuífero no confinado. Se muestran los mapas de isolíneas del nivel piezométrico y de la concentración en cloruros, los cuales indican la posición de la interfase.*

Palabras clave: acuífero costero, agua de mar, cuasi-tridimensional, intrusión, modelado.

VERSIÓN ABREVIADA EN CASTELLANO

Objetivos

El objetivo de este trabajo es presentar un modelo areal 2D basado en la aproximación de Dupuit para simular la intrusión marina en un acuífero costero no confinado. El modelo, que fue propuesto por Sorek et al. (2001), se resuelve utilizando un método de elementos finitos. La ecuación de transporte de solutos se puede estabilizar utilizando el método SUPG (Streamline Upwind Petrov Galerkin). El código propuesto nos permite calcular las isolíneas de la hidráulica y de la concentración de cloruros así como la profundidad de la interfase entre agua dulce y agua salada.

Introducción

Hay dos enfoques para el estudio de la intrusión en acuíferos costeros. El primero se denomina modelo de interfase neta y supone que los dos fluidos, agua dulce y agua salada, son inmiscibles y están separados por *una interfase bien marcada (Bear 1979; Larabi et al. 2001, Mahesha 2001; Kooi and Groen, 2001). La segunda se denomina modelo hidrodispersivo y tiene en cuenta la zona de transición entre el agua dulce y el agua salada (Bear, 1979; Bear et al., 2001; Simpson and Clement, 2003 ; Michel and Boufadel, 2000 ; Das and Datta, 2001 ; Voss and Souza,1987 ; Dierch and Kolditz, 2002).*

Este trabajo se fundamente en el segundo enfoque para simular la intrusión marina en un acuífero costero.

Metodología

Las ecuaciones del modelo areal 2D para la intrusión marina se expresan por Bear (1979) y Sorek et al. (2001) del siguiente modo.

- *Ecuación de flujo de densidad variable :*

$$\bar{\phi}.\varepsilon(h-b)\frac{\partial\bar{c}}{\partial t}+\frac{\bar{\rho}}{\bar{\rho}_0}\bar{\phi}\frac{\partial\bar{h}}{\partial t}=\nabla'\left\{(h-b)\frac{\bar{\rho}}{\bar{\rho}_0}\bar{k}\left[\nabla' h+\Omega(h-b)\varepsilon\nabla'\bar{c}\right]\right\}+\frac{\bar{\rho}}{\bar{\rho}_0}(h-b)(q_r-q_p)+N \qquad (1)$$

- *Ecuación de transporte:*

$$(h-b)\bar{\phi}\frac{\partial\bar{c}}{\partial t}+\bar{\phi}\bar{c}\frac{\partial h}{\partial t}-(h-b)q_r\bar{c}_r+(h-b)q_p\bar{c}_p=\nabla'[(h-b)\left(\bar{D}\nabla\bar{c}-\bar{u}\bar{c}\right)] \qquad (2)$$

Sorek et al. (2001) han propuesto la expresión generalizada de la descarga específica tal y como se muestra en la siguiente ecuación:

$$\bar{u}=-\bar{k}[\nabla' h+\Omega(h-b)\nabla'(\frac{\bar{\rho}}{\rho_s})] \qquad (3)$$

Con:

$$\Omega=0.5\theta+(1-\theta)\varpi \qquad (4)$$

θ=0.5 para el modelo de la interfase neta; θ=1 para el modelo de presión hidrostática.

La profundidad de la cuña salina (interfase neta) se calcula por (Sorek et al. (2001):

$$m=\varpi h+(1-\varpi)b \qquad (5)$$

Con:

$$\varpi=\frac{\bar{\rho}-\rho_0}{\rho_s-\rho_0} \qquad (6)$$

$$\rho=\rho_0(1+\varepsilon c)\ ,\quad \varepsilon=\frac{\rho_s-\rho_0}{\rho_0} \qquad (7)$$

$\bar{X}$ *: media de la variable X; ϕ: porosidad; h : nivel piezométrico de referencia; c : concentración normalizada,* c_p*,* c_r *: concentración de soluto extraído o inyectado; k : conductividad hidráulica;* q_r*,* q_p *: caudal extraído o inyectado; ρ : densidad del fluido con concentración c;* ρ_0 *: densidad del agua dulce;* ρ_s *: densidad del agua salada; ε: ratio de densidad; v : velocidad de Darcy;*

D_h *: tensor de dispersión hidrodinámica. En 2D su expresión está dado por*

$$D_{ij}=\alpha_T v\delta_{ij}+(\alpha_L-\alpha_T)\frac{v_i v_j}{|v|}+D_m\delta_{ij} \qquad (8)$$

α_L, α_T*: dispersividades longitudinal y transversal respectivamente.*

δ: función delta de Kronecker , i,j: índices de dimensión.
Con:

$$\overline{D} = \phi\overline{D}_h + \zeta(1-\Omega)\overline{c}(h-b)\varepsilon\overline{k} \quad (9)$$

$$\zeta = \frac{a_A}{a_A + \sqrt{a_L . a_T}} \quad (10)$$

Donde:
a_A : dispersividad aparente; a_L , a_T : dispersividades longitudinal y transversal respectivamente.

$\overline{D}_h$ *denota el tensor de la dispersión hidrodinámica promediada verticalmente y calculada por la ecuación (8).*

El modelo numérico de nuestro modelo matemático se basa en el método de elementos finitos. Seguidamente se aplicó el principio de Galerkin que consiste en minimizar las formas integrales de las ecuaciones (1) y (2) del siguiente modo:

$$\int_{\Re} w[\overline{\phi}.\varepsilon(h-b)\frac{\partial\overline{c}}{\partial t} + \frac{\overline{\rho}}{\overline{\rho}_0}\overline{\phi}\frac{\partial\overline{h}}{\partial t} - \nabla'\left\{(h-b)\frac{\overline{\rho}}{\overline{\rho}_0}\overline{k}\left[\nabla'\overline{h} + \Omega(h-b)\varepsilon\nabla'\overline{c}\right]\right\} - \frac{\overline{\rho}}{\overline{\rho}_0}(h-b)(q_r - q_p) - N]dR = 0 \quad (11)$$

$$\int_{\Re} \widetilde{w}(h-b)\overline{\phi}\frac{\partial\overline{c}}{\partial t} + \overline{\phi}\overline{c}\frac{\partial h}{\partial t} - (h-b)q_r\overline{c}_r + (h-b)q_p\overline{c}_p - \nabla'[(h-b)(\overline{D}\nabla\overline{c} - \overline{u}\overline{c})]dR = 0 \quad (12)$$

W=N para el método de Galerkín clásico.

$\widetilde{w} = N + p$ *para el método SUPG.*

Donde $p = \tau\, v\nabla N$ *es una difusión artificial o término de estabilización..*
τ : parámetro de estabilización dependiente del número de Peclet.
*v: velocidad de Darcy_*
El nivel piezométrico $\overline{h}$ y la concentración de soluto normalizada $\overline{c}$ se aproximan en el elemento finito por la expresión nodal:

$$\overline{h}(x,y,t) = \sum_{i=1}^{ne} N_i\overline{h}_i(t) \quad ; \quad \overline{c}(x,y,t) = \sum_{i=1}^{ne} N_i\overline{c}_i(t)$$

Se elige un modelo finito cuadrilateral.
$\overline{h}_i$, $\overline{c}_i$: valores de nivel piezométrico y concentración de soluto en los nodos.
N_i: función de forma del nodo i.
ne: número de nodos por elemento.

Reemplazando w por N y h, c por sus valores en las ecuaciones (11) y (12), integrando por partes utilizando el teorema de Green y teniendo en cuenta las condiciones de contorno sobre el elemento finito y armando la estructura de la malla se obtiene un sistema de la forma:

$$[A]\{U\} + [B]\left\{\frac{\partial U}{\partial t}\right\} + \{F\} = 0 \quad (13)$$

[A]: matriz de rigidez.
[B]: matriz de masas.
{F}: vector de condiciones de contorno más los términos relacionados $\overline{c}$ o $\overline{h}$
{U}: vector de incógnitas ($\overline{h}$ y $\overline{c}$)

Entonces para obtener los resultados de simulación se deben resolver el sistema de ecuaciones acopladas de flujo y transporte por linearización de Picard:

$$[A]_h\{\bar{h}\}+[B]_h\left\{\frac{\partial \bar{h}}{\partial t}\right\}+\{F\}_h=0 \quad (14)$$

$$[A]_c\{\bar{c}\}+[B]_c\left\{\frac{\partial \bar{c}}{\partial t}\right\}+\{F\}_c=0 \quad (15)$$

Aplicando el esquema implícito con los sistemas de flujo y transporte de solutos:

$$[\lambda[A]^{t+\lambda\Delta t}+\frac{1}{\Delta t}[B]^{t+\lambda\Delta t}]\{U\}^{t+\Delta t}=[(\lambda-1)[A]^{t+\lambda\Delta t}+\frac{1}{\Delta t}[B]^{t+\lambda\Delta t}]\{U\}^{t}-\{F\}^{t+\lambda\Delta t}=0 \quad (16)$$

Con $\lambda = 2/3$ se tiene el método de Galerkin y la solución converge rápidamente.

Hay varias formas de estabilización de parámetros dado τ. Se ha elegido el propuesto por Codina (2000): en régimen permanente:

$$\tau=\frac{1}{(2*\|v\|/\text{helem}+4\bar{\bar{D}}_h/\text{helem}^2)}$$

en regimen transitorio:

$$\tau=\frac{1}{(2*\|v\|/\text{helem}+4\bar{\bar{D}}_h/\text{helem}^2)}$$

Donde helem es la longitud característica del elemento a lo largo de la dirección de vector de velocidad.

Resultados y discusión

Régimen permanente

Para validar nuestro modelo numérico, se ha tratado el problema de Henry que es usado frecuentemente como un banco de pruebas en intrusión marina. Se ha considerado el ejemplo tratado por Bear et al. (2001). Se trata de un acuífero costero no confinado con las condiciones de contorno ilustradas en la figura 1. Los parámetros del problema se presentan en la tabla 1 y los resultados de la simulación se presentan en la Figura 2.

La figura 2 muestra los resultados obtenidos en régimen estacionario, después de 10 años sin recarga ni bombeos, en la sección Y = 250 m. Se muestran tres interfases, una calculada con nuestro modelo y las otras dos utilizando el método de Badon- Ghyben-Herzberg (BGH) y la solución de Glover. En ausencia de bombeos ($Q_w = 0$), hemos observado como la interfase de BGH, calculada con los valores de la piezometría (Bear, 1979), está cercana a la interfase calculada utilizando nuestro modelo cuando la profundidad es menor a 40 m. Por debajo de esta profundidad, las dos interfaces de BGH y Glover se mueven hacia dentro del acuífero. Coincidiendo con los resultados de Bear et al. (2001) obtenido con un modelo tridimensional, nosotros encontramos que la interfase obtenida con nuestros cálculos está cerca de la isolínea 0.5 de cloruros. El modelo de Ghyben-Herzberg sobreestima la posición neta del contacto agua dulce-agua salada.

Régimen transitorio

El segundo ejemplo es el mismo al tratado por Sorek et al. (2001). Se trata de dos zonas con diferentes parámetros y cada una contiene un hueco. El primer área, limitada por 0 <y <300, se caracteriza por $K_{xx} = K_{yy} = 1$ m/día, $\alpha_L = 10$, $\alpha_T = 1$, S_y=0.2. El sondeo localizado en (X = 500m, Y = 200m) proporciona un caudal de 750m³/día. La segunda área limitada por 300 < Y <1000, y con parámetros $K_{xx}= K_{yy} = 0.1$m/día, α_L =5m, $\alpha_T = 0.5$m, $S_y = 0.2$. El sondeo localizado en (X = 0, Y = 600) proporciona un caudal de 250m³/día. Los resultados de simulación después de 3650 días de bombeo se muestran en la figura 3.

Las figuras 3a y 3b muestran el estado del nivel piezométrico y la concentración en cloruro después de 3650 días de bombeo. Las líneas de isoconcentración de cloruros se mueven tierra adentro del acuífero en la

dirección del sondeo. Esto se debe a los altos valores de la velocidad debido al bombeo y al incremento del transporte por convección.

La figura 4 muestra la variación en el tiempo de la interfase agua dulce-agua salada calculada usando nuestro modelo en presencia de dos sondeos en la sección X=500m (sondeo cercano). La interfase se extiende hacia el agua como resultado de la depresión del bombeo con el tiempo. El avance del frente 100 m desde salinizado hasta la posición de partida se registra después de 10 años de bombeo y esto ocurre cerca del fondo del agua, mientras hay un progreso de 200 m cerca de la superficie.

Conclusiones

El código de ordenador desarrollado en FORTRAN se basa en la ecuación cuasi-tridimensional para simular la intrusión marina en acuíferos costeros. El modelo matemático desarrollado tiene en cuenta la interfase de agua dulce y agua salada (modelo hidro-dispersivo). Consiste en dos ecuaciones diferenciales parciales de flujo y transporte de solutos. La ecuación de transporte es parabólica-hiperbólica lo que hace difícil su resolución por el método clásico de Galerkin, lo que produce inestabilidades numéricas (oscilaciones), especialmente en condiciones donde domina la convección, esto es, cuando el número de Peclet excede el valor de dos. Se utiliza el método SUPG para resolver la ecuación de transporte y estabilizar la solución. Dicho método consiste en la adición de un término de difusión artificial al término de difusión para eliminar las oscilaciones. El término de estabilización seleccionado es el propuesto por Codina (2000). La posición de la interfase entre agua dulce y agua salada obtenida por nuestro modelo es representativa de la realidad al contrario que la interfase de Badon-Ghyben-Herzberg que sobreestima la zona de agua salada. Es más, es coincidente con la línea de isocloruros de 0.5 (Bear et al., 2001). La presencia de bombeos acelera el progreso de la interfase hacia el acuífero, particularmente en la vecindad del sondeo donde las isopiezas muestran un descenso en la piezometría.

Introduction

During the last ten years, many numerical models for seawater intrusion have been developed. These are based on two approaches, one based on the fresh-salt water sharp interface Bear (1979), Larabi *et al.*, (2001),Mahesha (2001), Kooi and Groen (2001) and the second based on the transition zone between fresh and saltwater (dispersion model) Bear (1979), Bear *et al.*, (2001), Simpson and Clement (2003), Michel and Boufadel (2000), Das and Datta (2001), Voss and Souza (1987), Dierch and Kolditz (2002), Huyakorn *et al.*, (1987), Molson and Frind (2002).

The simulation of sea water intrusion based on the sharp-interface model using the Badon Ghyben Herzberg approach (BGH) which supposes the two fluids are immiscible, are in static equilibrium and the transition zone is very thin to the fresh water zone (Bear, 1979). Cooper, (1959) observed that under certain conditions the thickness of the transition zone is wide which limitsthe application of this model. To simulate seawater intrusion in a coastal aquifer we apply the three-dimensional model, although as its calibration is difficult adding the stability of its numerical solution is not guaranteed.

Two-dimensional models exist based on the Dupuit formulation (Bear, 1979), Sorek *et al.,* (2001) and Zhoo and Chen (2000) which are able to take into account variable boundary conditions (recharge, sink…).

In this paper we have developed a model for a real simulation of sea water intrusion into a coastal aquifer. It consists of averaging 3D flow and transport equations, along the vertical direction over the saturated zone. The finite element method was applied with the Galerkin principle to discritize the flow equation and the Stream Upwind Petrov Galerkin (SUPG) method to discritize the transport equation. This method was first applied in fluid mechanics by Brooks and Hughes (1982), to solve the Navier Stokes equation The software elaborated in this work allows us to calculate the piezometric head and chloride concentration over the aquifer area. The results are compared to analytical solutions of GBH and Glover (Bear, 1979).

3D mathematical model

Seawater intrusion is governed by the coupling of the density-dependent flow equation with the piezometric head h and solute transport equation with the chloride concentration. These equations are expressed as follows (Bear, 1979):

- Flow equation:

$$S_0 \frac{\partial h}{\partial t} + \phi\varepsilon \frac{\partial c}{\partial t} - \frac{\rho}{\rho_0}\left(q_r - q_p\right) = \frac{1}{\rho_0} \nabla(\rho\phi v) \qquad (1)$$

- Solute transport equation :

$$\frac{\partial \phi c}{\partial t} - q_r c_r + q_p c_p = \nabla\left(\phi D_h \nabla c - \phi c v\right) \qquad (2)$$

- Darcy's flow:

$$\phi v = -K.(\nabla H + \frac{\rho - \rho_0}{\rho_0}\nabla_Z) \tag{3}$$

With:

$$\rho = \rho_0(1+\varepsilon c)\ ,\ \ \varepsilon = \frac{\rho_s - \rho_0}{\rho_0}$$

ϕ: porosity, S_0 : specific storage coefficient, h : reference head, c : normalized concentration, c_p, c_r : extracted or injected solute concentration, k :hydraulic conductivity, q_r, q_p :extract or injected fluid, ρ : fluid density with concentration c, ρ_0 : fresh water density, ρ_s : saltwater density. ε: density ratio coefficient , v : Darcy's velocity

D_h : hydrodynamic dispersion tensor , in 2D its expression is as following, Bear (1979)

$$D_{ij} = \alpha_T v\delta_{ij} + (\alpha_L - \alpha_T)\frac{v_i v_j}{|v|} + D_m\delta_{ij} \tag{4}$$

α_L , α_T :longitudinal and transversal dispersivities respectively

δ: symbol of Kronecker , i,j : dimension indices

2D mathematical model

To establish the areal mathematical model in (XY) we must average the integral of the terms of the two equations (1) and (2) on the vertical direction (Bear,1979).

For the flow equation we write:

$$\int_b^h S_0\frac{\partial h}{\partial t}dz + \int_b^h \phi\varepsilon\frac{\partial c}{\partial t}dz - \int_b^h \frac{\rho}{\rho_0}(q_r - q_p)dz = \int_b^h \frac{1}{\rho_0}\nabla(\rho\phi v)dz \tag{5}$$

We take into consideration the Leibnitz rule and denote

$$\bar{h} = \frac{1}{b-h}\int_b^h$$

the averaged hydraulic head on z direction. Then the equation (5) becomes:

$$\bar{\phi}.\varepsilon(h-b)\frac{\partial\bar{c}}{\partial t} + \frac{\bar{\rho}}{\bar{\rho}_0}\bar{\phi}\frac{\partial\bar{h}}{\partial t} = \frac{1}{\rho_0}\nabla'\{(h-b)(\bar{\rho}\bar{\phi}\bar{v})\} + \frac{\bar{\rho}}{\bar{\rho}_0}(q_r - q_p) + N \tag{6}$$

with:

$$\bar{\phi}\bar{v} = -\bar{k}\left[\nabla'\bar{h} + 0.5(h-b)\nabla'(\frac{\bar{\rho}}{\bar{\rho}_0})\right] \tag{7}$$

Equation (7) is the averaged Darcy's velocity on hydrostatic distribution (Sorek *et al.,* 2001).

This expression is not validated in the transition zone where the vertical component is not neglected, as we use the sharp interface model which allows us to write the density related to interface depth.

$$\bar{\rho}(h-b) = \rho_s(m-b) + \rho_0(h-m) \tag{8}$$

Then:

$$m = \varpi h + (1-\varpi)b \tag{9}$$

is the depth of the salt-fresh water interface with:

$$\varpi = \frac{\bar{\rho} - \rho_0}{\rho_s - \rho_0} \tag{10}$$

Similarly for velocity:

$$\bar{u}(h-b) = u_s(m-b) + u_f(h-m) \tag{11}$$

From equations (9), (10) and (11) the expression of Darcy's flow was found:

$$\bar{u} = -\bar{k}[\nabla'h + \varpi(h-b)\nabla'(\frac{\bar{\rho}}{\rho_s})] \tag{12}$$

Sorek *et al., (*2001) generalized this expression :

$$\bar{u} = -\bar{k}[\nabla'h + \Omega(h-b)\nabla'(\frac{\bar{\rho}}{\rho_s})] \tag{13}$$

with:

$$\Omega = 0.5\theta + (1-\theta)\varpi \tag{14}$$

θ=0.5 for the sharp interface model and θ=1 for the hydrostatic pressure model.

For establishment of the 2D transport equation we follow the same steps as the flow equation.

We calculate the integral of transport equation (2).

$$\int_b^h \frac{\partial\phi c}{\partial t}dz - \int_b^h q_r c dz = \int_b^h \nabla(\phi D_h\nabla c - \phi c v)dz \tag{15}$$

Denote

$$\bar{c} = \frac{1}{b-h}\int_b^h$$

averaged chloride concentration integral in vertical direction.

The 2D equation obtained by Sorek *et al.,* (2001) is:

$$(h-b)\bar{\phi}\frac{\partial\bar{c}}{\partial t} + \bar{\phi}\bar{c}\frac{\partial h}{\partial t} - (h-b)q_r\bar{c}_r + (h-b)q_p\bar{c}_p = \nabla'[(h-b)(\bar{D}\nabla\bar{c} - \bar{u}\bar{c})] \qquad (16)$$

is the 2D transport equation.

Where

$$\bar{D} = \phi\bar{D}_h + \zeta(1-\Omega)\bar{c}(h-b)\varepsilon\bar{k} \qquad (17)$$

In which $\bar{D}_h$ denotes the vertically averaged hydrodynamics dispersion tensor

ζ is factor associated with additional dispersion due to variations in the fluid horizontal velocities along vertical direction. Sorek *et al.,* (2001) have proposed a formulation of this factor based on the analytical solution of Cleary, Ungs (1978):

$$\zeta = \frac{a_A}{a_A + \sqrt{a_L.a_T}} \qquad (18)$$

Where a_A : apparent dispersivity

a_L , a_T : longitudinal and transversal dispersivities respectively

To sum up, the equations governing the areal dependent density flow and transport model for an unconfined aquifer are:

- Density dependent flow equation:

$$\bar{\phi}.\varepsilon(h-b)\frac{\partial\bar{c}}{\partial t} + \frac{\bar{\rho}}{\bar{\rho}_0}\bar{\phi}\frac{\partial\bar{h}}{\partial t} = \nabla'\left\{(h-b)\frac{\bar{\rho}}{\bar{\rho}_0}\bar{k}[\nabla'h + \Omega(h-b)\varepsilon\nabla'\bar{c}]\right\} + \frac{\bar{\rho}}{\bar{\rho}_0}(h-b)(q_r - q_p) + N \qquad (19)$$

Transport Equation:

$$(h-b)\bar{\phi}\frac{\partial\bar{c}}{\partial t} + \bar{\phi}\bar{c}\frac{\partial h}{\partial t} - (h-b)q_r\bar{c}_r + (h-b)q_p\bar{c}_p = \nabla'[(h-b)(\bar{D}\nabla\bar{c} - \bar{u}\bar{c})] \qquad (20)$$

With $\bar{u}$ and $\bar{D}$ are expressed by equations (13) and (17), respectively.

Numerical model

The standard Galerkin finite element method is used to discretize the fluid flow equation (19) but it cannot be applied to solve convection dominated transport problems.

The transport equation (20) is then solved using the Streamline Upwind Petrov-Galerkin method (SUPG) Codina (2000)The Galerkin approximation is obtained by minimizing the weighted integral form of equations (19) and (20). For the classical Galerkin method the weighting function w is the shape function N.Then the weighted integrals form of equations (19) and (20) are:

$$\int_{\Re} w[\bar{\phi}.\varepsilon(h-b)\frac{\partial\bar{c}}{\partial t} + \frac{\bar{\rho}}{\bar{\rho}_0}\bar{\phi}\frac{\partial h}{\partial t} - \nabla'\left\{(h-b)\frac{\bar{\rho}}{\bar{\rho}_0}\bar{k}[\nabla'\bar{h} + \Omega(h-b)\varepsilon\nabla'\bar{c}]\right\} - \frac{\bar{\rho}}{\bar{\rho}_0}(h-b)(q_r - q_p) - N]dR = 0 \qquad (21)$$

$$\int_{\Re} \tilde{w}(h-b)\bar{\phi}\frac{\partial\bar{c}}{\partial t} + \bar{\phi}\bar{c}\frac{\partial h}{\partial t} - (h-b)q_r\bar{c}_r + (h-b)q_p\bar{c}_p - \nabla'[(h-b)(\bar{D}\nabla\bar{c} - \bar{u}\bar{c})]dR = 0 \qquad (22)$$

W=N for classical Galerkin method°

$\tilde{w} = N + p$ for SUPG method

Where $p = \tau v \nabla N$ is artificial diffusion or stabilization term.

τ : stabilization parameter depended of Peclet number

v: Darcy's velocity

The hydraulic head $\bar{h}$ and normalized solute concentration $\bar{c}$ are approximated in finite element by nodal expression:

$$\bar{h}(x,y,t) = \sum_{i=1}^{ne} N_i\bar{h}_i(t) \quad ; \quad \bar{c}(x,y,t) = \sum_{i=1}^{ne} N_i\bar{c}_i(t)$$

A quadrilateral finite element is chosen

$\bar{h}_i$, $\bar{c}_i$: nodal values of hydraulic head and solute concentration

N_i: shape function of node i

ne: number of nodes per element

By replacing w by N and h, c by their values in equations (19) and (20), integrating by parts using Green's theorem and taking into account the boundary conditions on the finite element and after assemblage we obtain the follow system as the form:

$$[A]\{U\}+[B]\left\{\frac{\partial U}{\partial t}\right\}+\{F\}=0 \qquad (23)$$

[A]: stiffness matrix

[B]: mass matrix

{F}: vector boundary conditions +related terms c or h

{U}: vector of unknown variables ($\bar{h}$ and $\bar{c}$)

Sofor the flow equation U = $\bar{h}$:

$$[A]_h=\sum_e\int_{R^e}[\nabla'N]^T(h-b)\frac{\bar{\rho}}{\bar{\rho}_0}\bar{k}.[\nabla'N]dR^e$$

$$[B]_h=\sum_e\int_{R^e}[N]^T\frac{\bar{\rho}}{\bar{\rho}_0}\bar{\phi}.[N]dR^e$$

$$\{F\}_h=\sum_e\int_{R^e}[N]^T\bar{\phi}\varepsilon(h-b)..\frac{\partial\bar{c}^e}{\partial t}dR^e+\sum_e\int_{R^e}[\nabla'N]^T\bar{k}\varepsilon\Omega$$

$$(h-b)^2.\nabla'\bar{c}^e dR^e-\sum_e\int_{R^e}[N]^T\frac{\bar{\rho}}{\bar{\rho}_0}(h-b)(q_p^e-q_r^e)dR^e-$$

$$\sum_e\int_{\Gamma^e}[N]^T(\vec{n}.\frac{\bar{\rho}}{\bar{\rho}_0}\bar{k}(h-b)\nabla'h)d\Gamma^e-\sum_e\int_{\Gamma^e}[N]^T$$

$$(\vec{n}.\frac{\bar{\rho}}{\bar{\rho}_0}\bar{k}\varepsilon(h-b)^2\nabla'c)d\Gamma^e$$

For the transport equation U = $\bar{c}$ and c_p= $\bar{c}$

$$[A]_c=\sum_e\int_{R^e}\{\ [\nabla'N]^T(h-b)[\bar{D}].[\nabla'N]+\bar{\phi}\frac{\partial\bar{h}}{\partial t}[\tilde{w}]^T[N]+$$

$$[\tilde{w}]^T.(h-b).\left[\frac{\partial N}{\partial x}\right]\bar{u}_x^e+$$

$$[\tilde{w}]^T.(h-b).\left[\frac{\partial N}{\partial y}\right]\bar{u}_y^e+(h-b)q_p^e[\tilde{w}]^T[N]\ \}dR^e$$

$$[B]_c=\sum_e\int_{R^e}[\tilde{w}]^T(h-b)\bar{\phi}.[N]dR^e$$

$$\{F\}_c=\sum_e(-\int_{R^e}[\tilde{w}]^T(h-b)q_r^e c_r^e dR^e-\int_\Gamma[\tilde{w}]^T(\vec{n}.\bar{D}(h-b)\nabla'\bar{c})d\Gamma^e$$

After assembly we find the overall first-order no stationary and nonlinear coupled system.

$$[A]_h\{\bar{h}\}+[B]_h\left\{\frac{\partial\bar{h}}{\partial t}\right\}+\{F\}_h=0$$

$$[A]_c\{\bar{c}\}+[B]_c\left\{\frac{\partial\bar{c}}{\partial t}\right\}+\{F\}_c=0$$

The resolution of this system requires the use of iterative methods and a special time discretization which ensure the stability of the solution.

The resolution of the differential system describing the flow is not a stability problem and requires the usual time discretization schemes (Crank Nicolson) by cons when the convection term is added to the diffusion term, the transport equation resulting hybrid becomes parabolic - hyperbolic making a solution difficult or impossible by the Crank-Nicolson method. In this case the stability of the solution depends on the limitation of Peclet Pe and Courant Cr numbers. These two numbers should check the following Bear conditions (1979):

$$p_e=\frac{\|\vec{v}\|.helem}{D_h}\le 2;\quad C_r=\frac{\|\vec{v}\|.\Delta t}{helem}\le 1$$

Applying the implicit scheme on the fluid and solute transport systems

$$[\lambda[A]^{t+\lambda\Delta t}+\frac{1}{\Delta t}[B]^{t+\lambda\Delta t}]\{U\}^{t+\Delta t}=[(\lambda-1)[A]^{t+\lambda\Delta t}+$$

$$\frac{1}{\Delta t}[B]^{t+\lambda\Delta t}]\{U\}^t-\{F\}^{t+\lambda\Delta t}=0 \qquad (24)$$

With λ = 2 / 3 we find Galerkin method and the solution converges rapidly. There are several forms of parameter stabilization giving τ we take the one proposed by Codina (2000):

in steady state

$$\tau=\frac{1}{(2*\|v\|/helem+4\bar{\bar{D}}_h/helem^2)}$$

Transient:

$$\tau=\frac{1}{(1/(\lambda.\Delta t)+2*\|v\|/helem+4\bar{\bar{D}}_h/helem^2)}$$

with helem: characteristic length of the element along the direction of the velocity vector.

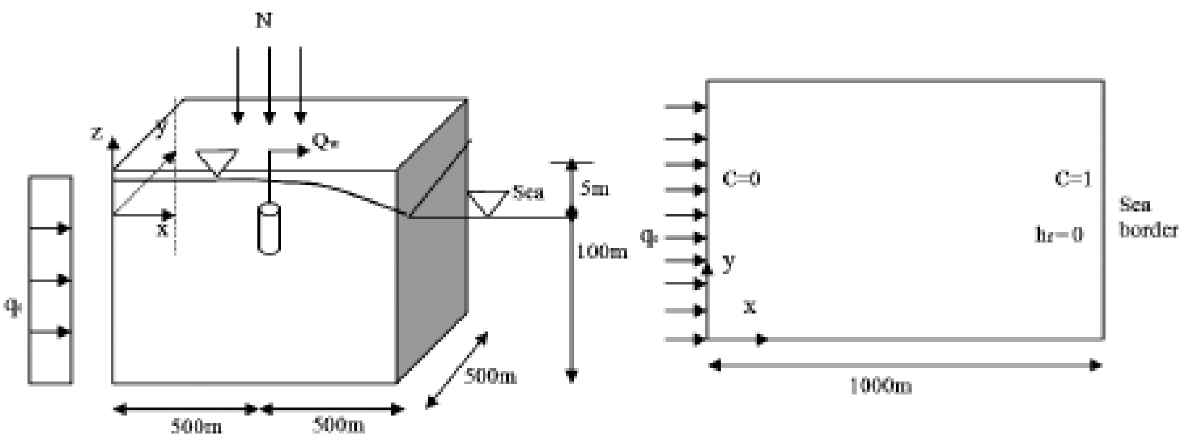

Figure 1. Domain and boundary conditions in the application of a phreatic coastal aquifer (Bear *et al.*, 2001)
Figura 1. *Dominio y condiciones de contorno en la aplicación a un acuífero costero no confinado ((Bear et al., 2001).*

Parameter	Value
Porosity (ϕ)	0.25
reference horizontal conductivity	20m/d
(Kxx =Kyy)	0.15m/y
natural replenishment (N)	0.0215m/y/unit of thick
lateral fresh water discharge (q_0)	
pumping rate (Qw)	392.2m³/d
density of pure freshwater (ρ_0)	1000 kg/m³
density of pure seawater (ρ_s)	1025 kg/m³
density coefficient (ε)	0.025
longitudinal dispersivity(α_L)	10m
transversal dispersivity (α_T)	1m
molecular diffusivity (D_m)	0 m²/d
finite element dimensions (Δx, Δy)	50m x 25m
apparent dispersivity a_A	3.9

Table 1. Problem parameters Bear *et al.* (2001).
Tabla 1. *Parámetros del problema de Bear* et al. *(2001).*

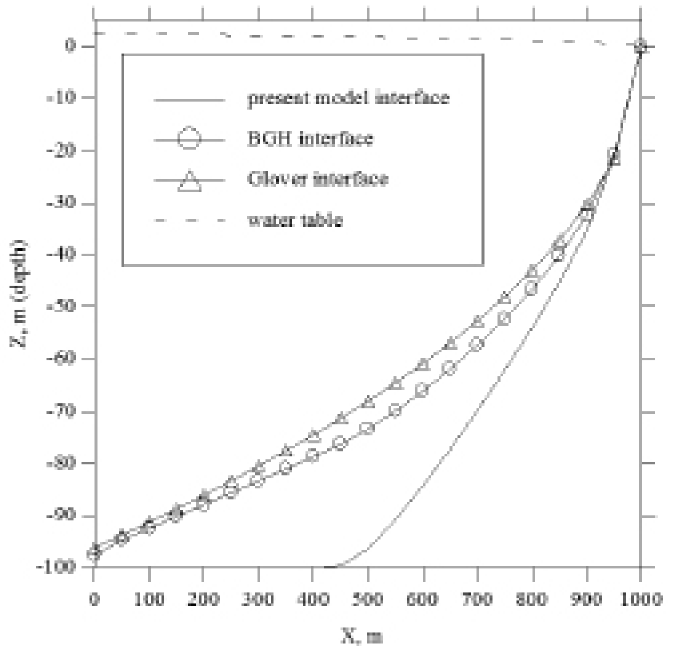

Figure 2. Steady state freshwater-saltwater interfaces at Y=250m.
Figura 2. *Interfases estacionarias de agua dulce - agua salada a Y = 250 m.*

Application to the case of a phreatic coastal aquifer

Steady state

In order to validate our numerical model, we have used the Henry problem, which is frequently used in the benchmarking of seawater intrusion.

We take again the same example used by Bear *et al.,* (2001). It is an unconfined coastal aquifer with boundary conditions illustrated in Figure 1.

The problem parameters are presented in Table 1 and the results of simulation are presented in Figure 2.

Transient case

The second example is the same treaty by Sorek *et al.,* (2001), it is a water table consisting of two zones of different parameters and each containing a hole. The first limited area 0 <y <300, characterized by $K_{xx} = K_{yy}$ = 1 m/day, α_L = 10, α_T = 1, Sy=0.2. The well located at (X = 500m, Y = 200m) provides a flow rate of 750m³ /day

The second area bounded by 300 <Y <1000, and whose parameters Kxx= kyy = 0.1m /day, α_L=5m, α_T = 0.5m, Sy = 0.2 the well located at (X = 0, Y = 600) provides a flow of 250m³/day, neglecting the sorption coefficient and the delay. Simulation results after 3 650 days of pumping are shown in Figure 3.

Results and discussion

Figure 2 shows the results obtained from steady state after 10 years without recharging and pumping in the section Y = 250 m. It shows three interfaces, one calculated by our model and the other two using the Badon-Ghyben Herzberg (BGH) approach and the solution of Glover. In the absence of a pumping rate (Qw = 0), we notice that the interface of BGH calculated from the values of the piezometric head (Bear 1979), is close to the interface calculated using our model when the depth is less than 40m. Beyond this depth the two interfaces of BGH and Glover move away into the aquifer. Compared to results of Bear *et al.,* (2001) obtained with a three-dimensional model; we find that the interface obtained by our calculation is close to the isochlor 0.5 line. The Ghyben Herzberg approach overestimates the fresh-salt water sharp position.

Figure 3(a) and (b) show the evolution of both the piezometric head and chlorine concentration in the water after 3 650 days of pumping. The isochloride concentration lines move toward the inland face of the aquifer in the direction of the well. This is due to the high values of velocity due to pumping and an increase in the solute transport by convection.

Figure 4 shows the variation in the time of the freshwater saltwater interface calculated using our model and in the presence of two wells in section X=500m (closed well). The interface extends into the water as a result of decrease in drilling activity over time. Advancement of the first 100m from salty to the starting position is recorded after 10 years of pumping and this is close to the bottom of the water, whilst we noted a progress of 200m near the surface.

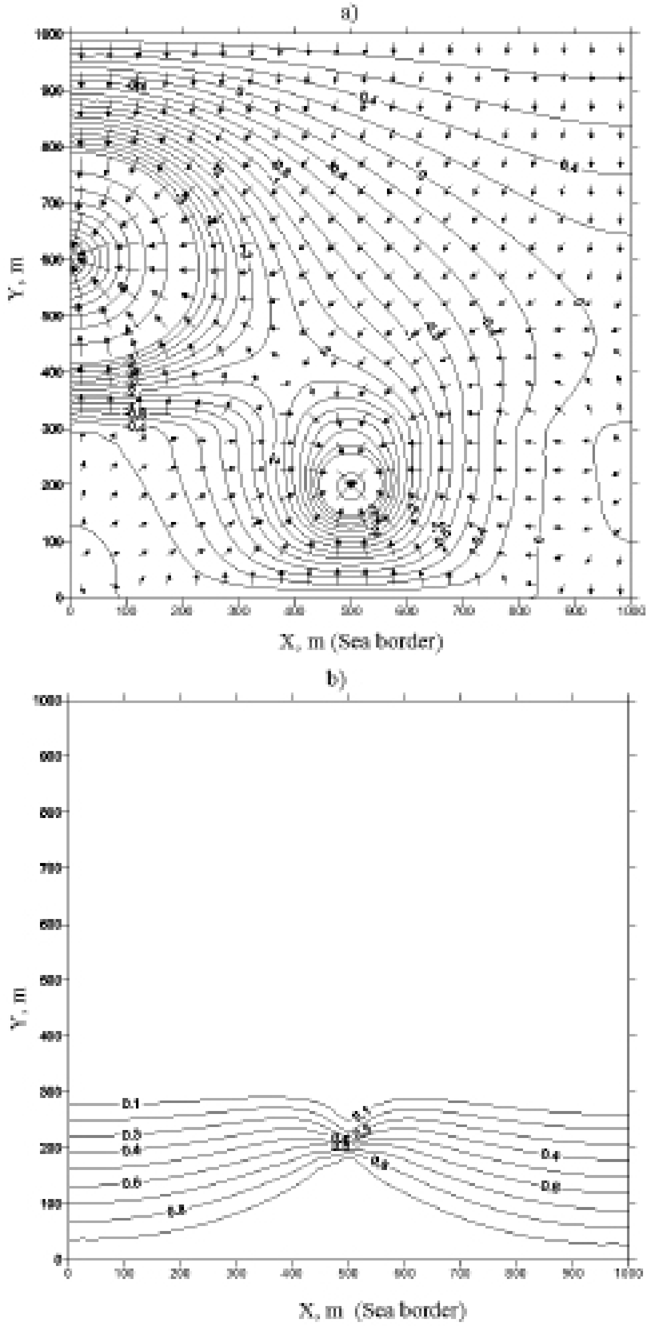

Figure 3. a: Piezometric head contours. b: Normalized chloride concentration contours.
***Figura 3**. a: Líneas isopiezas. b: Contornos de concentración normalizada de cloruro.*

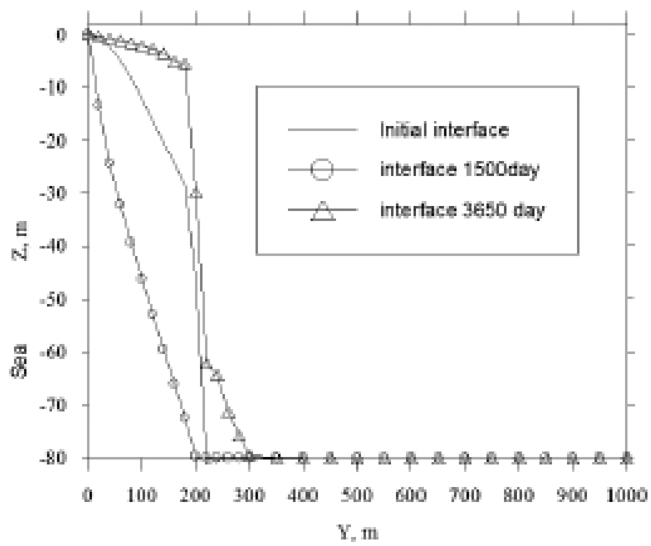

Figure 4. Temporal variation in freshwater –saltwater interface across section at X=500m.
***Figura 4**. Variación temporal en la interfase agua dulce - agua salada a lo largo de la sección a X= 500 m.*

Conclusions

The computer code developed in FORTRAN based on a quasi three-dimensional simulation and presented to simulate the seawater intrusion in coastal aquifers can achieve the same results found by applying a three-dimensional model. The mathematical model developed takes into account the coupling of salt water and fresh water (hydro-dispersive model). It consists of two partial differential equations of a fluid and the other the solute transport. The transport equation is both parabolic-hyperbolic which makes its resolution difficult by the classical Galerkin method of producing numerical instability (oscillations), especially in dominated convection, that is to say when the Peclet number exceeds two.

The SUPG method is used to solve the transport equation and stabilizes the solution. It consists of adding an artificial diffusion term to the diffusion term to eliminate oscillations. The term stabilizer selected is the same proposed by Codina (2000).

The position of the interface between freshwater and saltwater obtained by our model is representative of reality rather than the contrary of the Badon Ghyben Herzberg sharp interface, which gives an overestimation of the salt zone. Indeed, it coincides with the 0.5 isochlore, Bear *et al.,* (2001). The presence of a pumping well accelerates the progress of the interface to the interior of the aquifer, particularly in the vicinity of the well, where the isoheads also show a decrease in head.

References

Bear J. 1979. *Hydraulics of groundwater*, New York: McGraw Hill.

Bear, J., Zhoo, Q., Bensabet, J 2001. Three dimensional simulation of seawater intrusion in heterogeneous aquifer with application to the coastal aquifer of Israel. First international Conference on saltwater intrusion and coastal aquifers- Monitoring, Modelling and Management. Essaouira, Moroco, April 23-25.

Brooks, A.N., Hughes, T.J.R 1982. Streamline Upwind/ Petrov-Galerkin formulations for convection dominated flow with particular emphasis on the incompressible Navier-Stokes equations. *Computer Methods in Applied Mechanics and Engineering,* 32 , 199 – 259.

Cleary, R.w., Ungs, M 1978. Ground water pollution and hydrology mathematical models and computer program. Research report n°78-WR-15, Water resources program University Prinston, New Jersey.

Codina, R 2000. Stabilisation of incompressibility and convection through orthogonal subscales in finite elements method. *Computer Methods in Applied Mechanics and Engineering*, 190 (13- 14), 1579-1599.

Cooper, H.H.Jr 1959. A hypothesis concerning the dynamic balance of fresh water and salt water in a coastal aquifer, Journal of Geophysical Research, 64, 461- 467

Das, A., Datta, B 2001. Simulation of seawater intrusion in coastal aquifers: Some Typical responses. *Sadhana*, 26 (4), 317-352.

Diersch, H.J.G., Kolditz, O 2002. Variable density flow and transport in porous media: approaches and challenges. Advances in Water Resources, 25, 819-944.

Domenico, B 2010. Finite element solution to groundwater transport of solutes undergoing Decay and Non- Linear Sorption. *Hydrology Days,* pp. 12-13.

Huyakorn, P.S., Anderson, P.E , Mercer, J.W. et WHITE, H.O 1987. Saltwater intrusion in aquifers: development and testing of three-Dimensional finite element model. *Water Resources Research,* 23, 292-312.

Kooi, H., Groen, J 2001. Offshore continuation of coastal groundwater systems; predictions using sharp interface approximations and variable-density flow modelling. *Journal of Hydrology,* 246, 19-35.

Larabi, A., De Smed, F., Tanarhte, M 1997. Modelling saltwater intrusion by the finite elementMethod. Hydrochemistry (proceeding of the Rabat Symposium, April) IAHS Publ. n° 244.

Mahesha, A. 2001. Effect of strip recharge on sea water intrusion into aquifers Hydrologicalsciences- *Journal des Sciences Hydrologiques*, 46(2), 199-210.

Michel, C., Boufadel, A. 2000. Mechanistic study of nonlinear solute transport in a groundwater-surface system under steady state and transient Hydraulic conditions. *Water Resources Research,* 36 (9), 2549-2565.

Molson, J.W. and Frind, E.O. 2002. Saltflow version 3.0 Density – Dependent flow and mass transport model in three dimensions. User guide. University of Waterloo, Ontario, Canada.

Simpson, M.J., Clement, T.P. 2003. Theoretical analysis of the worthiness of Henry and Elder problems as Benchmarks of density-dependent groundwater flow models. *Advances in Water Resources*, 26, 17-31.

Sorek, S., Borisov, V.S, Yakirevich, A 2001. Two – Dimensional areal model for density dependent flow regime. *Transport in porous media*, 43,87-105.

Voss, Cl., Souza, WR 1987. Variable density flow and solute transport simulation of regional aquifers containing a narrow freshwater-saltwater transition zone. *Water Resources Research,* 23 (10), 1851-1866.

Zhoo, X., Chen, M 2000. Numerical simulation of sea water intrusion near Beihai, China. *Environnemental Geology,* 40 (1-2), 223-233.

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