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Probabilistic delineation of flood-prone areas based on a digital elevation model and the extent of historical flooding: the case of Ouagadougou

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ABSTRACT

The delineation of flood-prone areas is one of the most important steps for flood risk assessment and mitigation. This study uses a probabilistic and DEM-based framework for the delineation of flood-prone areas based on information about the extent of historical flooding in the area of interest. This is particularly useful for the delineation of flood-prone areas in cases where more accurate hydraulic profile calculations are not available. The delineation of flood-prone areas is carried out by using the Topographic Wetness Index (TWI) which allows for the delineation of a portion of a hydrographic basin potentially exposed to flooding by identifying all the areas characterized by a topographic index that exceeds a given threshold. A Bayesian updating framework is used for estimating the TWI threshold for identifying the flood-prone areas based on available information on the spatial extent of historical flooding. An application of the proposed method is demonstrated for the delineation of potentially flood-prone areas in the city of Ouagadougou, based on the observed spatial extent of the 2009 flooding event in the city.

Key words: flood-prone areas, Topographic Wetness Index, Bayesian parameter estimation, GIS, Africa.

Delineación probabilística de áreas inundables basada en modelos digitales de elevaciones e inundaciones históricas: el caso de Ouagadougou

RESUMEN

La delineación de áreas propensas a sufrir inundaciones es uno de los pasos más importantes para la evaluación del riesgo de avenidas y su mitigación. Este estudio utiliza un enfoque probabilístico basado en el modelo digital de elevaciones para la delineación de áreas propensas a inundarse utilizando información sobre la extensión de avenidas históricas en el área de interés. Esto es particularmente útil para la delineación de áreas propensas a inundarse en los casos donde no están disponibles los cálculos de perfiles hidráulicos más precisos. La delineación de áreas propensas a la inundación se lleva a cabo utilizando el índice topográfico de humedad (TWI) que permite la delineación de una porción de la cuenca hidrográfica potencialmente expuesta a inundación mediante la identificación de todas las áreas caracterizadas por un TWI que excede un determinado umbral. Un enfoque de actualización Bayesiana es utilizado para estimar el umbral de TWI que permite identificar las áreas propensas a inundarse basado en información disponible sobre la extensión espacial de las inundaciones históricas. Se muestra una aplicación del método propuesto para delinear áreas potencialmente inundables en la ciudad de Ougadougou, sobre la base de la observación de la extensión espacial de las inundaciones de 2009 en dicha ciudad.

Palabras clave: África, áreas propensas a la inundación, estimación bayesiana de parámetros, índice topográfico de humedad, GIS, TWI.

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Introducción y metodología

La delineación de áreas propensas a inundarse es uno de los primeros pasos en una metodología integrada para la planificación urbana y la gestión de riesgos. Es conocido que la delineación de las áreas susceptibles de sufrir inundaciones no solo proporciona información muy útil para los gestores administrativos sino que también es de gran utilidad como información de apoyo para indicar la futura dinámica urbana y sus tendencias. Este trabajo emplea un enfoque basado en modelos digitales de elevaciones (MDEs) para la delineación de áreas susceptibles a la inundación, identificadas por el índice topográfico de humedad (TWI). El TWI permite la delineación de una porción de una cuenca hidrográfica potencialmente expuesta a inundación por avenidas de agua mediante la identificación de todas las áreas caracterizadas por un índice topográfico que excede un determinado umbral. El umbral TWI depende de la resolución del MDE disponible y de la la topología de la cuenca hidrográfica (esto es, urbano, peri-urbano o rural). Asumiendo que el territorio urbano se puede caracterizar por un único umbral, se puede calibrar este umbral basándose en los resultados de la delineación detallada del perfil de inundación para zonas seleccionadas. El TWI tiene una interpretación puramente topográfica (básicamente mide la capacidad para acumular agua) y se calcula fácilmente en un entorno de Sistemas de Información Geográfica (GIS) para áreas muy extensas. Por otra parte, los cálculos hidráulicos precisos pueden requerir la propagación bidimensional de avenidas, siendo más adecuadas (en términos de esfuerzo computacional) para áreas de extensión limitada (micro-escala, por ejemplo 1:2000). En un trabajo previo de los autores, el umbral TWI se ha calibrado basándose en información disponible de cálculos de los perfiles hidráulicos para un área limitada dentro de la cuenca.

En este estudio, se ha adoptado el mismo enfoque metodológico introducido en Jalayer et al. (2014) para calibrar el umbral TWI basado en la información histórica disponible sobre la extensión espacial de las inundaciones en el área de interés. Se puede considerar la información proporcionada por la extensión espacial de diferentes inundaciones históricas a través de la inferencia bayesiana. El enfoque bayesiano permite la caracterización probabilística del umbral mediante el cálculo de la probabilidad complementaria de la falsa delineación de zonas propensas a la inundación como una función de varios valores de umbral (Figura 1). La probabilidad de la falsa delineación se calcula como la suma de la probabilidad de indicar una zona como susceptible de inundación cuando en realidad no es indicada como tal por los datos de la extensión histórica, y la probabilidad de que una zona sea señalada como no propensa a la inundación pero indicada como que sí es propensa por los datos de la extensión histórica. Aplicando el procedimiento previamente indicado, teniendo en cuenta toda la información disponible acerca de la extensión histórica de inundación para varias zonas dentro de la cuenca, se obtiene la distribución de probabilidad para el valor del umbral TWI.

La novedad en el enfoque propuesto radica en el uso de la estimación bayesiana de parámetros para la caracterización de incertidumbres en la delineación de áreas propensas a la inundación. Más específicamente, estas incertidumbres están relacionadas con la evaluación del umbral del TWI. El proceso del calibrado del umbral se ha hecho en este trabajo basándose en la extensión espacial de eventos de inundaciones previas (Figura 2).

Este esfuerzo de investigación se ha efectuado dentro del programa marco europeo FP7 titulado Cambio Climático y Vulnerabilidad Urbana (CLUVA) en África. La idea principal fue el desarrollo de una metodología que es aplicable para la delineación de áreas urbanas susceptibles de sufrir inundaciones en un contexto africano. Como ilustración se han delineado las áreas urbanas propensas a la inundación en la ciudad de Ouagadougou (Burkina Faso). En este caso los umbrales TWI se calibran basándose en la extensión espacial observada en las inundaciones que la ciudad sufrió en 2009.

Resultados y Conclusiones

Las áreas propensas a las inundaciones basadas en el método TWI se han identificado a través de la delineación de áreas que destacan por un índice TWI mayor que un cierto umbral. Este umbral se puede calibrar basándose en la información disponible, tal como los perfiles de inundación calculados para una cierta área dentro de la cuenca o la extensión espacial de las avenidas acaecidas en eventos históricos (Figura 2). Un método basado en los MDEs y en estimación Bayesiana de parámetros (Figuras 3-6) se ha utilizado para calcular la estimación de la máxima verosimilitud a posteriori y los percentiles 16^{th} y 50^{th} para el umbral TWI en base a la extensión de las avenidas históricas para varias zonas dentro del área de interés (Tabla 1). El calibrado del umbral TWI se ha evaluado cuantitativamente mediante el cálculo de un índice definido en la ecuación (10). Los valores resultantes para el índice I se han listado en la Tabla 2. El mapa TWI resultante (Figura 7) se puede utilizar como proxy para áreas potencialmente propensas a la inundación y puede ser superpuesto con varias bases de datos geo-espaciales teniendo información morfológica y de exposición con el fin de identificar puntos con riesgo de inundación especialmente alto.

Esta metodología se ha aplicado para delinear las áreas potencialmente propensas a la inundación en la ciudad de Ouagadougou en Burkina Faso, basada en información acerca de dos áreas inundadas por el evento de inundaciones de 2009. La distribución de probabilidad resultante para el umbral TWI va a producir varios valores estimados de umbral TWI que pueden ser usados para escalar el mapa hasta abarcar una mayor área. En la utilización de estos resultados es importante enfatizar que la incertidumbre del umbral TWI lleva a una diferencia considerable en la extensión estimada de las áreas afectadas y consecuentemente en la exposición a inundación.

El procedimiento de acualización Bayesiana que se propone en este trabajo ha sido muy útil en la incorporación de la información espacial procedente de ambas áreas. Es más, se ha comprobado que el procedimiento posibilita la identificación de varios estadísticos del umbral TWI (por ejemplo el percentil 50, la máxima verosimilitud, el umbral del 99% de máxima verosimilitud) basado en la información disponible sobre el conjunto de inundaciones históricas. Este es un resultado importante si se considera que el umbral TWI se calibra generalmente basándose en información precisa procedente de los perfiles de inundación de una ventana espacial más pequeña. En otras palabras, tener la posibilidad de calibrar el umbral TWI basándose en la extensión de avenidas históricas es un método muy eficiente para detectar las áreas potencialmente propensas a la inundación sin tener la necesidad de realizar cálculos hidráulicos previos.

Habría que destacar que la aplicación del método TWI (con un umbral fijo) asume implícitamente que el área entera de interés se puede caracterizar por el mismo umbral TWI. Por otra parte, el procedimiento bayesiano propuesto tiene la capacidad de actualizar la distribución de probabilidad del umbral TWI basándose en diferentes áreas dentro de la zona de interés. Si las predicciones sucesivas del umbral (por adicción de más información de nuevas áreas) demuestran un incremento significativo en la dispersión, uno puede re-examinar la hipótesis de que todo el área se puede identificar con un umbral TWI único.

Introduction

Delineation of flood-prone areas is one of the first steps in an integrated methodology for urban planning and risk management. Arguably, the delineation of flood-prone areas not only provides useful information for the policy makers but it can also be useful as support information for indicating future urban dynamics and trends (De Risi and Jalayer 2013). This study uses a DEM-based framework for delineation of flood-prone areas, identified by the Topographic Wetness Index (TWI) (Qin et al. 2011). TWI allows for the delineation of a portion of a hydrographic basin potentially exposed to flood inundation by identifying all the areas characterized by a topographic index that exceed a given threshold. The TWI threshold depends on the resolution of the available Digital Elevation Model (DEM) and the topology of the hydrographic basin (i.e., urban, peri-urban or rural) (Manfreda *et al.,* 2008, Manfreda *et al.*, 2011). Assuming that the urban territory can be characterized by a single threshold, this threshold can calibrated based on the results of a detailed delineation of the inundation profile for selected zones. The TWI has a purely topographic interpretation (it basically measures the capability to accumulate water) and it is quite straightforward to calculate very large areal extensions in the GIS environment. On the other hand, accurate hydraulic calculations may involve two-dimensional flood propagation and are more suitable (in terms of computational effort) for limited areal extensions (micro-scale, e.g., 1:2000). In a previous study by the authors, the TWI threshold was calibrated based on information available from calculation of the hydraulic profiles for a limited area within the basin (Jalayer *et al.,* 2014).

In this study, the same methodological approach introduced in Jalayer *et al.,* (2014) has been adopted for calibrating the TWI threshold based on the historical information available on the spatial extent of flooding in the area of interest. Different spatial extent historical flooding information can be considered in the proposed methodology through Bayesian inference. The Bayesian framework enables probabilistic characterization of the threshold by calculating the complementary probability of false delineation of flood-prone zones as a function of various threshold values. The probability of false delineation is calculated as the sum of the probability of indicating a zone as flood prone, whilst it is not indicated as such by the historical extent, and the probability that a zone is indicated as not flood prone but is indicated as flood prone by the historical extent. Applying the above-mentioned procedure, taking into account all available information about the extent of historical flooding for various zones within the basin, leads to a probability distribution for the TWI threshold value.

In a traditional approach, the flood-prone areas are usually identified based on available historical flooding data or by defining a buffer zone around the rivers (see Gall *et al.*, 2007 or Apel *et al.,* 2009 for a comprehensive discussion on identification of flood-prone areas). A recent work by Degiorgis *et al.,* (2012) uses pattern classification techniques for the delineation of flood-prone areas and hazard graduation within these areas based on remote-sensed data.

The novelty of the approach we propose lies in the use of Bayesian parameter estimation for the characterization of uncertainties in delineating the potential flood-prone areas. More specifically, these uncertainties are related to the evaluation of the threshold for the TWI. The threshold calibration process done in this work is based on available flooding spatial extent from previous flooding events.

This research has been conducted within the European FP7 project entitled "Climate Change and Urban Vulnerability (CLUVA) in Africa". The main idea was to develop a method that could be applicable for the delineation of urban flood-prone areas in the African context. As a demonstration, flood-prone areas were delineated for the city of Ouagadougou (Burkina Faso). In this case the TWI thresholds were calibrated based on the observed spatial extent of the 2009 flooding event in the city.

DEM-based delineation of flood-prone areas

Delineation of flood-prone areas using TWI

The TWI, initially introduced by Kirkby (1975), has been shown to be strongly correlated to the area exposed to flood inundation (Manfreda *et al.*, 2007, 2008, 2011). The TWI for a given point *O* within the hydrographic basin is calculated as follows:

$$TWI = \log\left(\frac{A_s}{\tan\beta}\right) \tag{1}$$

where A_s is the specific catchment area expressed in metres and calculated as the local up-slope area draining through a generic point of interest per unit contour length (A/L); β is the local slope at the point in question expressed in degrees.

The TWI allows for the delineation of a portion of a hydrographic basin potentially exposed to flooding (referred to herein as flood prone or more briefly as FP) by identifying all the areas characterized by a topographic index that exceeds a given threshold. The TWI threshold depends on the resolution of the DEM and the topology of the hydrographic basin. This threshold is usually calibrated based on the results of detailed delineation of the inundation profile for selected zones (e.g., see De Risi et al., 2013). The method presented in this paper for the estimation of TWI threshold is based on information other than the inundation profile. Herein, approximate maps delineating the flood-prone areas (e.g., based on historical information) are used instead of the inundation profile for calibrating the threshold.

Maximum likelihood estimation of the TWI threshold

The delineation of flood-prone areas is strictly dependent on the TWI threshold, assuming that the urban territory can be characterized by a single threshold value. This section describes how the likelihood function for the TWI threshold is calculated based on the historical spatial extent for a selected zone of interest within the basin.

Let W represent the spatial window of a zone of interest (within the basin) containing historical flooding extent information. Moreover, let FP represent the flood-prone areas identified as TWI>τ and IN represent the inundated areas based on historical information, where τ is the TWI threshold. Figure 1 (a) illustrates W in a schematic manner and the portions identified as FP and IN.

The probability of the correct delineation of the flood-prone areas or the likelihood function for the TWI threshold τ denoted as L(τ |*W*) for various values of τ can be calculated as follows:

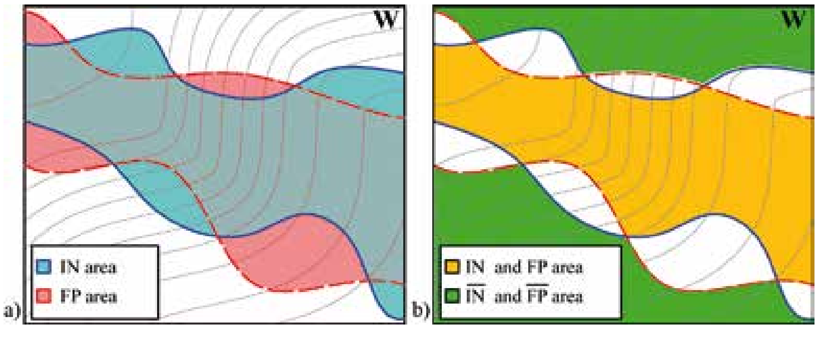

Figure 1. A schematic representation of the spatial window of reference (W), flood-prone (FP) and inundated (IN) areas.

Figura 1. *Una representación esquemática de la ventana espacial de referencia (W), y de las áreas propensas a las inundaciones (FP) e inundadas (IN).*

$$L(\tau|W)=P(FP,IN|\tau,W)+P(\overline{FP},\overline{IN}|\tau,W) \quad (2)$$

where P(FP, IN|τ,*W*) denotes the probability that a given point within zone *W* is identified both as flood-prone FP (using the TWI method) and inundated IN (based on historical flooding spatial extent), and conditioned on (the | sign) a given value of τ of the TWI threshold. The area FP and IN is indicated by orange colour in Figure 1 (b). Similarly, P ($\overline{FP},\overline{IN}$|$\tau$,*W*) denotes the probability that a given point within the zone of interest is neither identified as FP nor as IN conditioned on a given value of τ of the TWI threshold. The areas that are neither FP and nor IN is indicated by green colour in Figure 1 (b). It can be shown that the likelihood function in Eq. 2 can be equivalently expressed as the complement of probability of false delineation of flood-prone areas or the so-called *cry-wolf*:

$$L(\tau|W)=1-P(FP,\overline{IN}|\tau,W)-P(\overline{FP},IN|\tau,W) \quad (3)$$

where P($FP,\overline{IN}$|τ,*W*) for a given point within zone *W* denotes the probability that it is indicated as FP by the TWI method but not IN, based on historical information. Vice versa, P ($\overline{FP},IN$|τ,*W*) for a given point within zone *W* denotes the probability that it is not indicated as FP but it results as inundated in the delineation of the inundated zones.

Going back to Eq. 2, the terms $P(FP,IN|\tau,W)$ and $P(\overline{FP},\overline{IN}|\tau,W)$ can be expanded, using the probability theory's product rule (Jaynes 2003), as follows:

$$P(FP,IN|\tau,W)=P(FP|\tau,W)\cdot P(IN|FP,\tau,W) \quad (4)$$

$$P(\overline{FP},\overline{IN}|\tau,W)=P(\overline{FP}|\tau,W)\cdot P(\overline{IN}|\overline{FP},\tau,W) \quad (5)$$

where the term $P(IN|FP,\tau,W)$ for a given point denotes the probability of being IN, given that it is identified as FP and $P(\overline{IN}|\overline{FP},\tau,W)$ denotes the probability of not being IN conditioned or not being FP, given the threshold value τ. The terms $P(FP|\tau,W)$ and $P(\overline{FP}|\tau,W)$ represent the probability of being FP or not being FP respectively, given the TWI threshold value τ.

Estimation of the likelihood function using the areal extents

The micro-scale estimations: Let $A_W(FP)>\tau$ (the extent of the portion in red in Figure 1 (a)) denotes the areal extent of the flood-prone portion of the zone *W* identified via the TWI method (note that $A_W(FP)$ is a function of τ since the flood-prone areas are identified as areas with TWI) and $A_W(IN)$ the areal extent of the inundated potion of *W* indentified via historical flooding spatial extent (the extent of the portion in blue in Figure 1(a)). Analogously, $A_W(\overline{FP})$ and $A_W(\overline{IN})$ refer to the areas of the not flood-prone and not inundated portions, respectively. The probability terms $P(IN(T_R)|FP,\tau,W)$ and $P(\overline{IN}|\overline{FP},\tau,W)$ can be estimated by the ratio of areal extents, as expressed in the following:

$$P(IN|FP,\tau,W)=\frac{A_W(IN,FP)}{A_W(FP)} \quad (6)$$

$$P(\overline{IN}|\overline{FP},\tau,W)=\frac{A_W(\overline{IN},\overline{FP})}{A_W(\overline{FP})} \quad (7)$$

where $A_W(IN,FP)$ denotes the areal extent of the portion of the area *W* that is both FP and IN (the extent of the area colored as orange in Figure 1 (b)); $A_W(\overline{IN},\overline{FP})$ denotes the areal extent of the portion of the area *W* that is neither FP nor IN (the extent of the area coloured as green in Figure 1 (b)). As mentioned above, the areal extent ratios $A_W(IN,FP)$, $A_W(\overline{IN},\overline{FP})$, $A_W(FP)$ and $A_W(\overline{FP})$ are, by definition, all functions of the TWI threshold τ.

The meso-scale estimations: In the previous section, it was demonstrated how $P(IN|FP,\tau,W)$ and $P(\overline{IN}|\overline{FP},\tau,W)$ have been estimated using the areal extent ratios calculated in a micro-scale delineated by window *W*. However, $P(FP|\tau,W)$ and $P(\overline{FP}|\tau,W)$ also need to be estimated in order to be able to calculate the likelihood function. We have chosen to estimate the above two terms using the areal extent ratios calculated in the meso-scale (city-scale)[1]. Denoting the total administrative area of the city under consideration as A_{urban} and denoting the total areal extent within the city having TWI greater than the given threshold τ as $A_{urban}(FP)$, the term $P(FP|\tau)^2$ can be estimated as:

$$P(FP|\tau)=\frac{A_{urban}(FP)}{A_{urban}} \quad (8)$$

1. Note that this term could have also been estimated based on the information contained within window W. However, we chose to use the whole extent of the city as a reference. Therefore, in this case the information provided by the inundation profiles within W was not used. In fact, the term P(FP|τ,W) for simplicity is referred to as P(FP|τ) hereafter.

2. The conditioning on W is left out for brevity and simplicity of formulations.

The probability $P(not\,FP\,|\,\tau)$ can then be calculated as $1-P(FP\,|\,\tau)$.

Finally, the likelihood function in Eq. 2 can be calculated by substituting the terms calculated in Eqs.6, 7 and 8 for Eqs. 4 and 5 and adding together these two last equations. The maximum likelihood estimate for the TWI threshold can be calculated as the τ value that maximizes the likelihood function in Eq. 2.

Using Bayesian parameter estimation to estimate τ based on information from more than one spatial window

If we suppose that some background information is available on the value of the TWI threshold τ. , the maximum likelihood method presented in the previous section can be extended to a Bayesian parameter estimation, where the available background information is represented by a *prior* probability distribution. That is, the *posterior* probability distribution for τ given the information provided by the inundation profile within the spatial window *W* can be expressed as[3]:

$$p(\tau\,|\,W)=\frac{L(\tau\,|\,W)p(\tau)}{\sum L(\tau\,|\,W)p(\tau)} \tag{9}$$

where $p(\tau\,|\,W)$ denotes the posterior probability distribution for τ given spatial window *W*; $L(\tau\,|\,W)$ is the likelihood function for τ calculated in the previous section and $p(\tau)$ is the prior probability distribution for τ before having the information on the inundation profile for window *W*. Note that Eq. 9 is particularly useful for calculating the threshold τ when we have historical flooding spatial extents for more than one spatial window within the basin (note that they could also correspond to different flooding events). In this case, the posterior probability $p(\tau\,|\,W_1)$ can be used as prior probability distribution in order to calculate the posterior probability distribution $p(\tau\,|\,W_1,W_2)$ considering both spatial windows W_1 and W_2 (see for example Figure 2 (a)) and so on and so forth.

The case of Ouagadougou

On September 1, 2009, an unprecedented deluge of rain hit the capital city of Ouagadougou and wide-spread damage was caused (destruction of buildings and infrastructure). More than 25 cm of rainfall in 12 hours turned the streets of Ouagadougou into fast-flowing rivers. The infrastructure was severely affected as the floods cut off electricity, fresh water and fuel supplies. The city is used to heavy seasonal rainfall but this was the worst flooding in 50 years. An estimated 109.000 people were left homeless. In the Bayesian framework described in this paper, the areal extent of this flooding event is used, in order to delineate the flood-prone areas for Ouagadougou.

Delineation of flood-prone areas for Ouagadougou using the TWI

The TWI is calculated in the GIS framework by applying Eq. 1 and based on the DEM of the city (vertical resolution: 3 meters). Figure 2 (a) illustrates the resulting TWI map for Ouagadougou. It can be observed that the TWI values vary between 8 and 25; in particular, the highest TWI values can be identified around the natural water channels.

The inundation spatial extent

The threshold for Ouagadougou has been calibrated based on the inundated area of the 2009 flooding event, Figure 2 (b). Based on the information available at the internet site (http://www.mapaction.org/map-catalogue/mapdetail/1719.html), it was possible to geo-reference the previous image and generate a spatial dataset. As can be depicted from Figure 2 (b), two inundated areas have been identified. In the following section, the smaller area is called A1 and the bigger area is referred to as A2.

Bayesian estimation of the TWI threshold

In this section, we demonstrate how the procedure described herein can be applied to calculate the likelihood of a correct identification of the flood-prone areas as a function of the TWI threshold. In this case, Bayesian parameter estimation is adopted based on information about the spatial extent of historical flooding from more than one spatial window. In particular, the likelihood function calculated based on spatial window W1 is used as a-priori probability distribution for the calibration of the TWI threshold for window W2. The historical areal extents A1 and A2 (for spatial windows W1 and W2, respectively) are approximately equal to 29.9 km^2 and 10.6 km^2, respectively. The two spatial windows W1 and W2, which contain the areas A1 and A2, have aerial extents of about 156 km^2 and

3. Strictly speaking, the formulation in Eq. 9 should have been conditioned on the "correct identification" of the flood-prone areas for window W (see Eq. 2). However, for the sake of simplicity and tractability of the equations, we have used the symbol W in order to imply in, a concise manner, all the additional information about the inundation profile contained within window W.

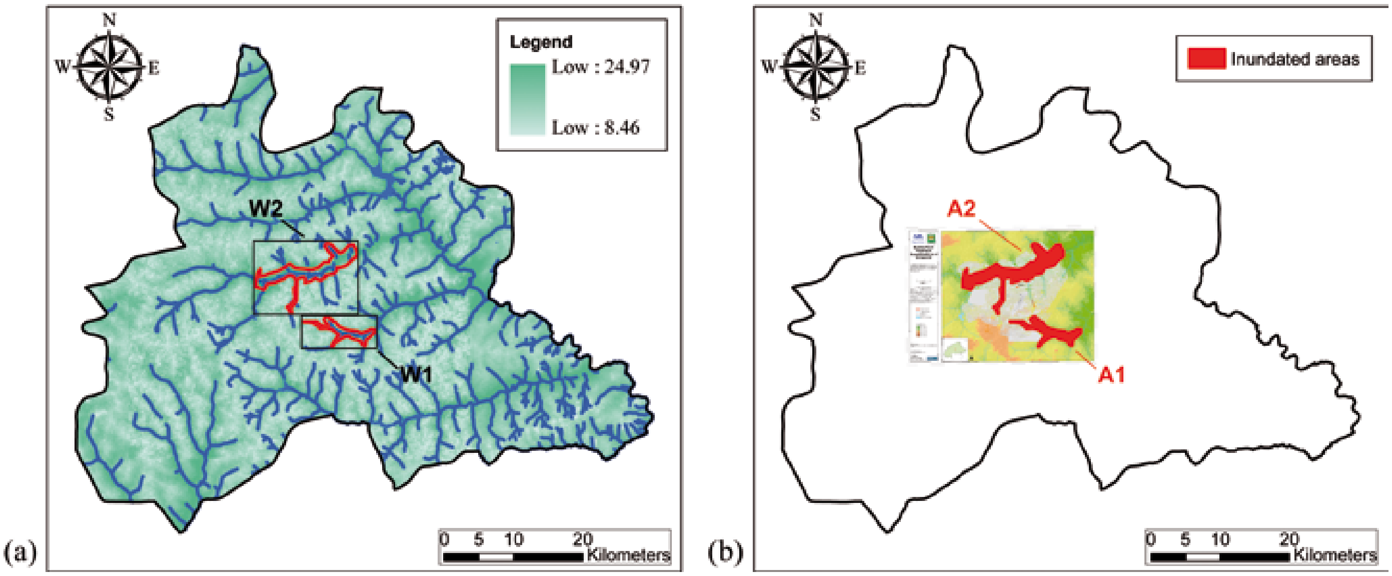

Figure 2. (a) TWI for Ouagadougou, W1 and W2 the two case study windows (b) Inundated areas after the 2009 events, A1 (the smaller) and A2 (the larger).
Figura 2. *TWI para Ouagadougou, W1 y W2 son las dos ventanas de casos de estudio, (b) Áreas inundadas después de los eventos de 2009, A1 (la pequeña) y A2 (la mayor).*

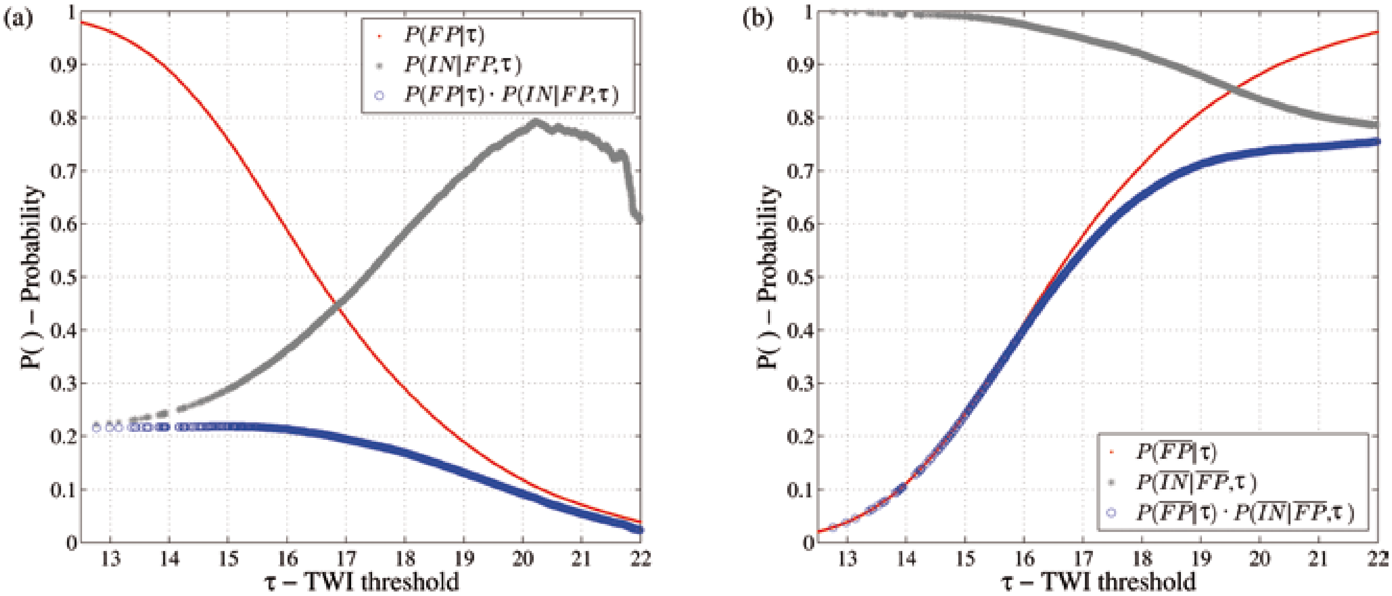

Figure 3. (a) Probability of being FP and IN given τ. b) Probability of being $\overline{FP}$ and $\overline{IN}$ given τ (for spatial window W1).
Figura 3. *(a) Probabilidad de ser FP e IN dado τ. (b) Probabilidad de ser $\overline{FP}$ y $\overline{FP}$ dado τ (para la ventana espacial W1).*

49 km^2, respectively. The total area of the greater Ouagadougou area is equal to around A_{urban}=3 046 km^2.

Information from W1: In the first stage, the Bayesian method described here is applied to calibrate the TWI threshold for the first spatial window W1 from Eq. 9. Since there is no prior information available about the TWI threshold, the prior probability distribution p(τ) is assumed to be a uniform distribution. The likelihood function L(τ|W1) is calculated from Eq. 2. For all the possible values of τ, the probability that a given zone is flood prone, denoted by P(FP|τ), is calculated from Eq. 8 (based on meso-scale information) and is plotted as red dots in Figure 3 (a). Moreover, the probability P(IN|FP,τ) that a given point is inundated, given that it is already indicated as FP for a given value of τ, is calculated from Eq. 6 (based on micro-scale estimations) and plotted as grey stars in Figure 3(a). The probability P(FP,IN|τ) that a given point is indicated both as flood prone (by the TWI method) and inundated (based on the hydraulic profile) is calculated as the product of P(FP|τ) and P(IN|FP,τ) in Eq. 4 and is plotted as blue circles in Figure 3 (a). In a similar manner, the probability that a given point is not indicated as flood prone (based on the TWI method) is calculated as the complementary probability of being flood prone in Eq. 8 and is plotted as red dots in Figure 3 (b). The probability that a given zone is not indicated as inundated, given that it is not flood prone, for a given value of τ, is calculated from Eq. 7 and is plotted as the grey stars in Figure 3 (b). Finally, the probability that a given point is not inundated and not flood prone, for a given value of τ, is calculated from Eq. 5 and is plotted as the red circles in Figure 3 (b). The likelihood function for threshold τ is finally calculated from Eq. 2 by adding together the probability of being flood prone and inundated and the probability of not being flood prone and not being inundated, for all possible τ values (i.e., adding together the curves illustrated by blue circles in Figure 3 (a) and 3 (b)). Figure 4 (a) illustrates the likelihood function L(τ|W1) for τ based on historical flooding extent information from window W1. It is worth noting that L(τ|W1) is equal to the posterior probability distribution function for τ from Eq. 9 as the prior distribution is uniform. Figure 4 (b) illustrates the (posterior) cumulative distribution function for τ based on information from W1.

Information from W1 and W2: In the next step, the posterior distribution for τ based on information coming from W1 (Figure 4 (a)) is used as prior distribution p(τ) in Eq. 9 for calibrating the TWI threshold based on historical flooding extent information from W2. The same as described for W1, Figure 5 (a) plots the probability that a given zone is flood prone P(FP|τ) (based on meso-scale estimations) as red dots. The probability P(IN|FP,τ) is calculated from Eq. 6 (based on micro-scale estimations) and plotted as grey stars. The probability P(FP,IN|τ), calculated from Eq. 4 as the product of P(FP|τ) and P(IN|FP,τ), is plotted as blue circles. The probability that a given point is not indicated as flood prone (based on the TWI method) is calculated as the complementary probability of being flood prone in Eq. 8 and is plotted in Figure 5 (b) as red dots. The probability that a given zone is not indicated as inundated given that it is not flood prone, for a given value of τ, is calculated from Eq. 7 and is plotted as grey stars in Figure 5 (b). Finally, the probability that a given point is not inundated and not flood prone, for a given value of τ, is calculated from Eq. 5 and is plotted as red circles in Figure 5 (b). The likelihood function for threshold τ is finally calculated from Eq. 2 by

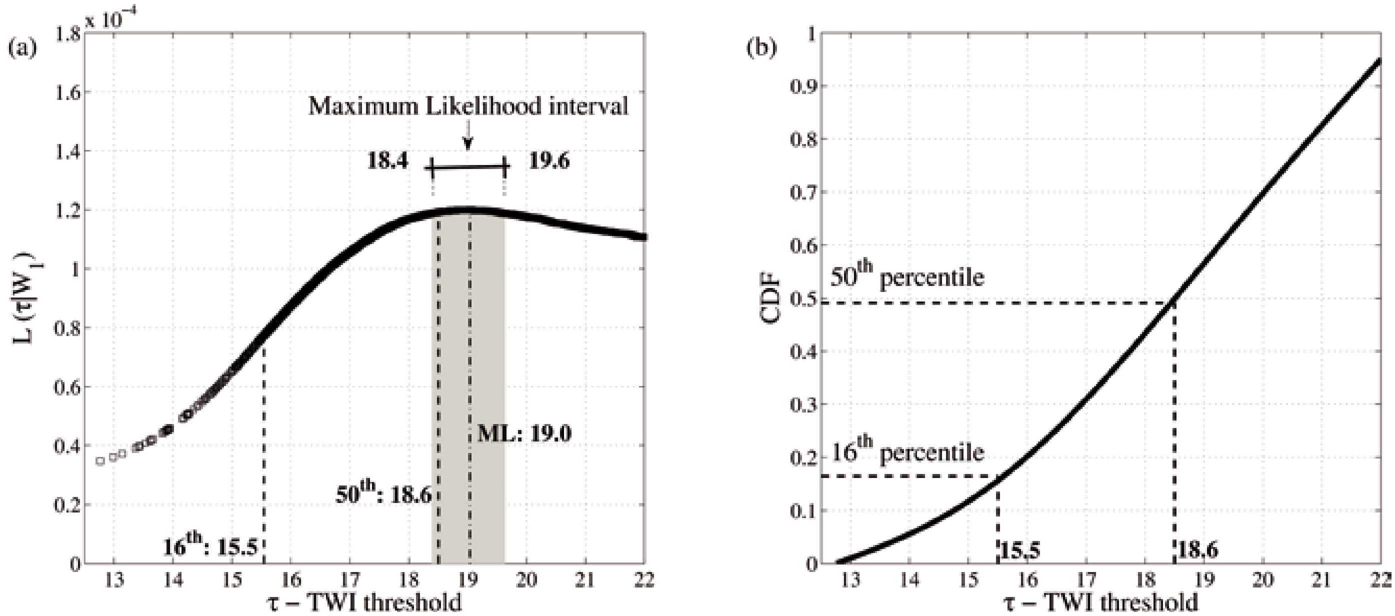

Figure 4. Bayesian parameter estimation (a) the likelihood function L(τ|W_1), b) posterior cumulative density function (CDF) for the TWI threshold based on the information from W_1 .
Figura 4. *Estimación Bayesiana de parámetros. (a) la función de verosimilitud L(τ|W_1), (b) función de densidad de acumulada (CDF) a posteriori para el umbral TWI basado en la información de W_1.*

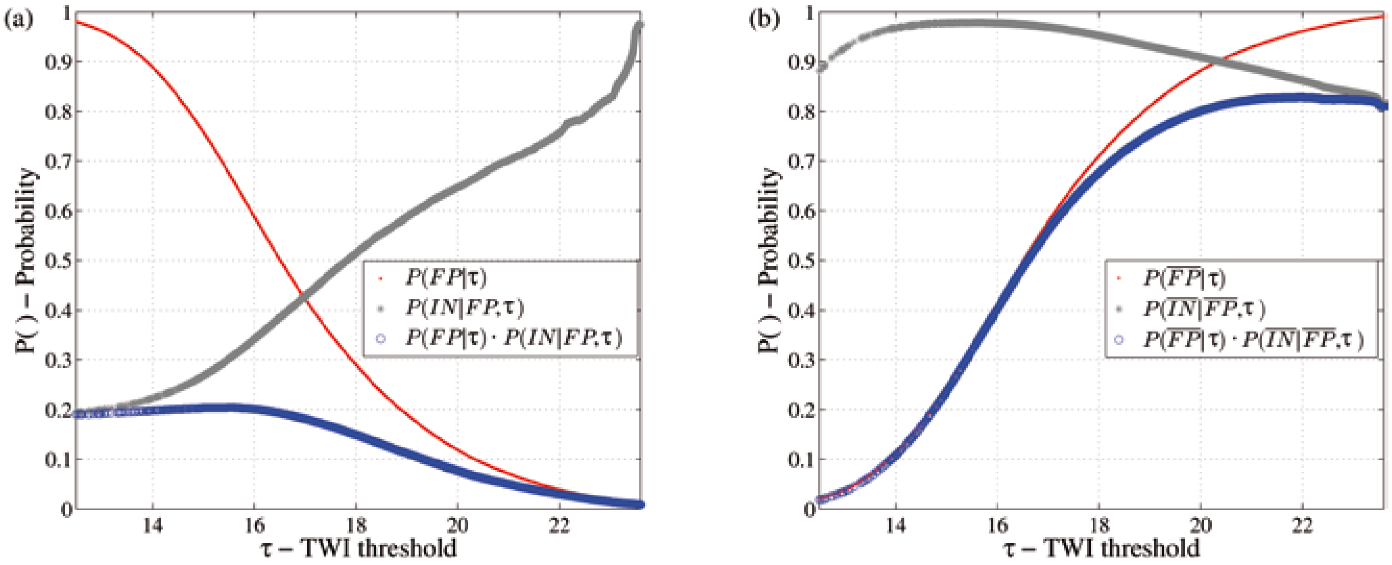

Figure 5. (a) Probability of being FP and IN given τ. b) Probability of being $\overline{FP}$ and $\overline{IN}$ given τ (spatial window W2).
Figura 5*. (a) Probabilidad de ser FP e IN dado τ. (b) Probabilidad de ser* $\overline{FP}$ *y* $\overline{FP}$ *dado τ (para la ventana espacial W2).*

summing up the probability of being flood prone and inundated and the probability of not being flood prone and not being inundated, for all possible τ values (i.e., summing up the curves illustrated by blue circles in Figure 5 (a) and 5 (b)). Figure 6 (a) illustrates the posterior probability distribution for τ based on historical flooding extent information from window W2 and W1 calculated from Eq. 9. The figure also illustrates the prior probability distribution p(τ) or L(τ|W1) based on information only from W1. Figure 6 (b) illustrates the (posterior) cumulative distribution function for τ denoted as p(τ|W1,W2) based on information from W1 and W2. Figure 6 (b) illustrates the (posterior) CDF for τ together with the threshold values corresponding to 16th and 50th percentiles of the probability distribution equal to 16.6, and 19.1, respectively.

Consequently, the maximum likelihood estimate for τ (i.e., the value that corresponds to the maximum likelihood from the posterior probability distribution in Figure 6 (a)) can be identified as τ=19.5. Furthermore, by identifying the τ values corresponding to more than 99% of the maximum likelihood value, it is possible to define a maximum likelihood interval, which varies between τ^-_{ML}=18.9 and τ^+_{ML}=19.9. This is, from a practical point of view, the information used for calibrating the TWI threshold leads to the identification of a maximum likelihood interval [18.9, 19.9] for τ. Table 1 reports the statistics of the TWI threshold for both stages of the calibration process for W1 and W2. It can be observed that integrating the historical extent information from the second window (in addition to the first one) leads to a slight increase in the TWI threshold and a slight decrease in the length of the 99% maximum likelihood interval. This implies that the information coming from areas Area 1 and Area 2 does not provide evidence against the working hypothesis of homogeneity in the TWI threshold between these two areas. Ideally, if historical flood extent information were available for other areas in the greater Ouagadougou area, this homogeneity hypothesis could have been further verified for the entire city.

Table 1 reports the threshold values that are calculated with the proposed procedure.

	τ_{ML}	τ_{16}	τ_{50}	τ^-_{ML}	τ^+_{ML}
W_1	19.0	15.5	18.6	18.4	19.6
W_1, W_2	19.5	16.6	19.1	18.9	19.9

Table 1. The statistics for the *TWI* threshold distribution for the case of Ouagadougou
Tabla 1. *Estadísticos para la distribución del umbral TWI para el caso del Ouagadougou.*

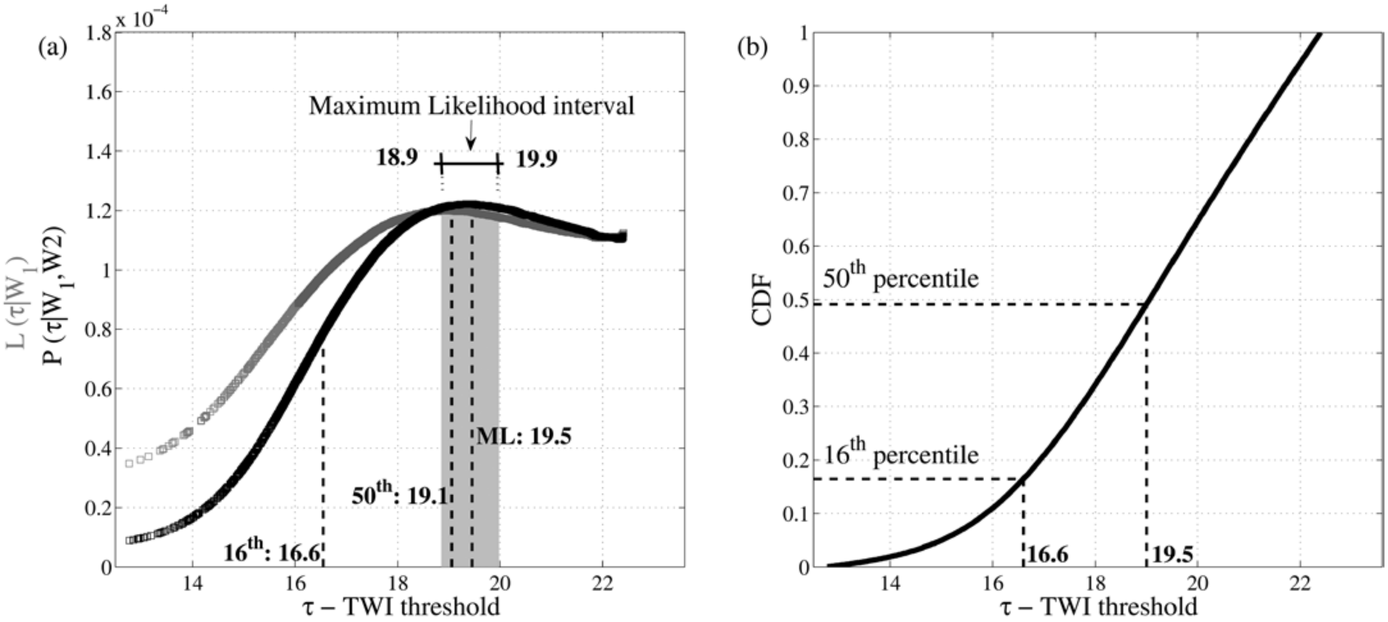

Figure 6. Bayesian parameter estimation (a) the likelihood function P(τ|W_1,W_2), b) posterior cumulative density function (CDF) for the TWI threshold based on the information from W_1 and W_2.
Figura 6. *Estimación Bayesiana de parámetros. (a) la función de verosimilitud P(τ|W_1,W_2), b) función de densidad acumulada (CDF) a posteriori para el umbral TWI basado en la información de W_1 and W_2.*

The corresponding (up-scaled) map of the flood-prone area for the entire city of Ouagadougou is shown in Figure 7 (a). It is possible to observe in Figure 7 (b) that the *TWI* map with a threshold equal to the maximum likelihood *TWI* threshold τ_{ML} matches the spatial extent of the areas inundated by the 2009 flooding event.

In order to quantitatively evaluate the TWI threshold calibration, the following indicator ratio is defined over a prescribed window W:

$$I = \frac{A_W\left(IN, FP\right) + A_W\left(\overline{IN}, \overline{FP}\right)}{A_W} \tag{10}$$

The term $A_W\left(IN, FP\right.$ is the areal extent that is identified both by the hydraulic profile contour *IN* and the hazard zonation *FP*. The term denoted by $A_W\left(\overline{IN}, \overline{FP}\right)$ is the areal extent that is outside both $IN(h, T_R)$ and *FP* contours. The subscript *W* denotes the spatial window of interest. It should be noted that the closer the *I* ratio is to unity, the more accurate the mapping of flood-prone areas is going to be. The value of *I* for W1 and W2 is reported below.

Concluding remarks

The flood prone areas based on the TWI method are identified by delineating the areas distinguished with a TWI larger than a certain threshold. This threshold can be calibrated based on available information, such as inundation profiles calculated for a certain area within the basin or the delineated spatial extent of the flooding based on historical flooding events. A DEM-based method based on Bayesian parameter estimation is used for calculating the posterior maximum likelihood estimate and the 16th and 50th percentiles for the TWI threshold, based on historical

	I
W_1	0.86
W_1, W_2	0.86

Table 2. The analytical check index.
Tabla 2. *El índice analítico de chequeo.*

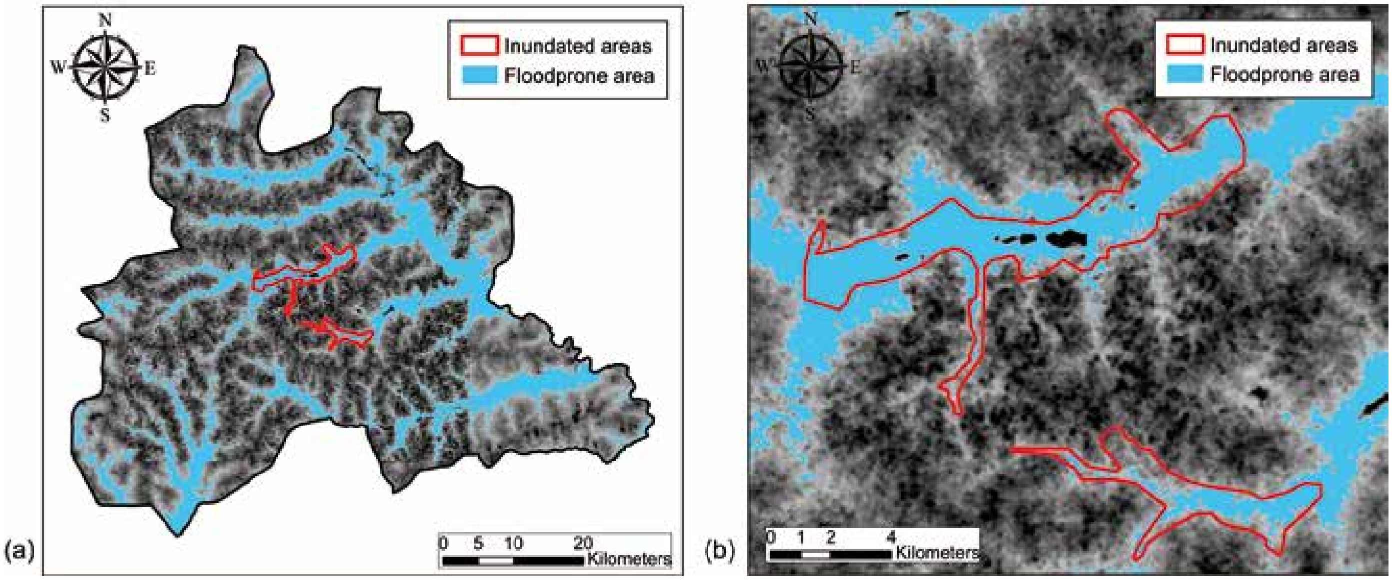

Figure 7. (a) Up-scaling the flood prone areas for the entire city; b) matching for the flood-prone area (FP) and the inundation extent (IN) for the spatial window of interest.
Figura 7. *(a) Agregación de las áreas propensas a la inundación para toda la ciudad; (b) correspondencia de las áreas propensas a la inundación (FP) y la extensión inundada (IN) para la ventana espacial de interés.*

flooding extent available for various zones within the area of interest. The resulting TWI map can be used as a proxy for potentially flood-prone areas and can be overlaid with various geo-spatial datasets with morphological and exposure information in order to identify the flooding risk hotspots (see Jalayer *et al.,* in press).

This method is applied to delineate the potentially flood-prone areas for the city of Ouagadougou, Burkina Faso, based on information about two areas inundated by the flooding event of 2009. The resulting probability distribution for the TWI threshold leads to various TWI threshold estimates that can be used to up-scale the map into a larger areal extent. When acting on these results, it is important to emphasize that uncertainty in the TWI threshold leads to a considerable difference in the estimated areal extent of affected areas and consequently the exposure to flooding.

The Bayesian updating procedure we propose has been quite useful in incorporating the spatial information from both areas. Moreover, it can be observed that the procedure manages to indentify various statistics of the TWI threshold (e.g., 50th percentile, maximum likelihood, the 99% maximum likelihood threshold) based on information available from the extent of historical flooding This is an important outcome if we consider that the TWI threshold is usually calibrated based on information coming from an accurate calculation of the inundation profile in a smaller spatial window. In other words, having the possibility of calibrating the TWI threshold based on the extent of historical flooding makes it a very efficient method for screening the potentially flood-prone areas without having the necessity to perform accurate hydraulic calculations beforehand.

We should mention that the application of the TWI method (with a fixed threshold) implicitly assumes that the entire area of interest can be characterized by the same TWI threshold. The Bayesian procedure proposed has the capability of updating the probability distribution for TWI threshold based on the various areas within the zone of interest. If the successive predictions of the threshold (by adding more information from new areas) demonstrate a significant increase in the dispersion, the hypothesis that the whole area can be identified with a unique TWI threshold might be reexamined.

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